

# Automatised full one-loop renormalisation of the MSSM I: The Higgs sector, the issue of $\tan\beta$ and gauge invariance

N. Baro<sup>1)</sup>, F. Boudjema<sup>1)</sup> and A. Semenov<sup>2)</sup>

*1) LAPTH, Université de Savoie, CNRS,  
BP 110, F-74941 Annecy-le-Vieux Cedex, France*

*2) Joint Institute of Nuclear Research, JINR, 141980 Dubna, Russia*

## Abstract

We give an extensive description of the renormalisation of the Higgs sector of the minimal supersymmetric model in **SloopS**. **SloopS** is an automatised code for the computation of one-loop processes in the MSSM. In this paper, the first in a series, we study in detail the non gauge invariance of some definitions of  $\tan\beta$ . We rely on a general non-linear gauge fixing constraint to make the gauge parameter dependence of different schemes for  $\tan\beta$  at one-loop explicit. In so doing, we update, within these general gauges, an important Ward-Slavnov-Taylor identity on the mixing between the pseudo-scalar Higgs,  $A^0$ , and the  $Z^0$ . We then compare the  $\tan\beta$  scheme dependence of a few observables. We find that the best  $\tan\beta$  scheme is the one based on the decay  $A^0 \rightarrow \tau^+\tau^-$  because of its gauge invariance, being unambiguously defined from a physical observable, and because it is numerically stable. The oft used  $\overline{\text{DR}}$  scheme performs almost as well on the last count, but is usually defined from non-gauge invariant quantities in the Higgs sector. The use of the heavier scalar Higgs mass in *lieu* of  $\tan\beta$  though related to a physical parameter induces too large radiative corrections in many instances and is therefore not recommended.

# 1 Introduction

Were it not for the radiative corrections to the lightest Higgs mass [1], the minimal supersymmetric model or MSSM would have been a forgotten elegant model a long time ago. Indeed, at tree-level the mass of the lightest Higgs is predicted to be less than the mass of the  $Z^0$  boson,  $M_{Z^0}$ . That would have been a real pity from a model whose most appealing and foremost motivation was to solve the hierarchy problem and make the Higgs more natural, beside providing a very good Dark Matter candidate. The renormalisation of the Higgs sector of the MSSM is therefore important. It is also important because it provides a link to the other parameters of the Standard Model, namely all the masses of the particles. It also encodes another parameter that can be seen to describe the relative scale of the two vacuum expectation values needed for each Higgs doublet of the SM, often referred to as  $\tan\beta$  and which permeates all the other sectors of the MSSM: the gaugino/higgsino sector and the sfermion sector. Many renormalisation schemes or definitions of this parameter are unsatisfactory, as we will see, mainly because they lack a physical interpretation or do not correspond to a physical and gauge independent parameter.

The aim of this paper is to give an extensive description of the renormalisation of the Higgs sector in **SloopS** at one-loop. **SloopS** is a fully automated code for the one-loop calculation of any cross section or decay in the MSSM at one-loop. Although there have been a few studies of the renormalisation of the Higgs sector, (see [2, 3] for a recent review of the Higgs in supersymmetry) some performed even beyond the one-loop approximation especially as concerns the mass of the lightest CP-even Higgs [4, 5], looking at the problem afresh while keeping the issue of gauge invariance in mind, will prove rewarding. Moreover our motivation in developing **SloopS** was also to have a *full* one-loop renormalisation of all the sectors of the MSSM in a coherent way and therefore the study of the Higgs sector is a first step. We will point at the non gauge invariance of some definitions of  $\tan\beta$ . Although this has been known, see for example [6], and pointed out at two-loop in the usual linear gauge [7], most practitioners have kept the usage of some non-gauge invariant definitions of  $\tan\beta$  because of their simplicity at the technical level being based on definitions involving two-point function self-energies. With the automatisation of the loop calculations, considerations and definitions of  $\tan\beta$  based on three-point functions (decays) are hardly more involved than those based solely on two-point functions describing self-energies, including transitions.

In the approach adopted within **SloopS**, we strive for an on-shell, OS, renormalisation scheme in particular for  $\tan\beta$ . We rely on a general non-linear gauge fixing constraint to make the gauge parameter dependence of different schemes for  $\tan\beta$  at one-loop explicit. In so doing we rederive and update the Ward-Slavnov-Taylor identity on the  $A^0 Z^0 / H^\pm W^\pm$  mixing in the non-linear gauge. We then compare qualitatively and quantitatively the  $\tan\beta$  scheme dependence of a few observables.  $A^0$  is the CP-odd Higgs scalar and  $H^\pm$  are the charged Higgses. We find that the best  $\tan\beta$  scheme is the one based on the decay  $A^0 \rightarrow \tau^+ \tau^-$  because of its gauge invariance, being unambiguously defined from a physical observable, and because it is numerically stable. The oft used  $\overline{\text{DR}}$  scheme performs almost as well on the last count, but is usually defined from non-gauge invariant quantities in the Higgs sector. The use of the heavier CP-even scalar Higgs mass in *lieu* of  $\tan\beta$  though related to a physical parameter induces too large radiative corrections in many instances and is therefore not acceptable. It has been argued that the definitions within the Higgs sector may be considered universal compared to a definition involving a particular Higgs decay for example. However, as stressed in [8], staying within the confines of the Higgs sector and the Higgs potential, one faces the issue that many definitions may be basis dependent, as we will see this will translate at one-loop into issues about gauge invariance for these definitions. As concerns the application to the corrections to the lightest Higgs mass our one-loop treatment is certainly not up-to-date, however our motivation is to stress the gauge dependence issues and compare the impact of the scheme dependence for  $\tan\beta$  for many observables starting for those directly related to the properties of the Higgses of the MSSM, before reviewing in our forthcoming studies [23] the

impact of  $\tan\beta$  on observables in the charginos/neutralinos as well as the sfermion sectors. We feel that this issue is of importance as is a consistent one-loop OS implementation.

The present paper is organised as follows. In Section 2 we review the Higgs sector of the MSSM at tree-level. This may, by now, be considered trivial but it is a necessary step before we embark on the renormalisation procedure. We also detail this part in order to show what might qualify as a physical basis independent observable. Section 3 presents the non-linear gauge fixing condition that we use. This includes 8 gauge fixing parameters which are crucial in studying many issues related to gauge invariance that are not easily uncovered when one works within the usual linear gauge. Section 4 constitutes the theoretical core of our analyses and deals with renormalisation, introducing counterterms for the Lagrangian parameters and the field renormalisation constants. We expose our renormalisation conditions and update the Slavnov-Taylor identities involving the  $A^0 - Z^0$  and  $H^\pm - W^\pm$  transitions. Section 5 is devoted to defining  $\tan\beta$ . We consider a few schemes. Before turning to applications and numerical results we briefly describe how our automatic code is set-up in Section 6. In Section 7 a numerical investigation of the scheme dependence and gauge dependence of these schemes is studied taking as examples loop corrections to Higgs masses, decays of the Higgses to fermions and to gauge and Higgs bosons. Section 8 gives our conclusions. The paper contains two appendices. Appendix A details the derivation of Slavnov-Taylor identity for the  $A^0 - Z^0$  transition. Field renormalisation may be introduced at the level of the *unphysical* fields before rotation to the physical fields is performed, Appendix B relates these field renormalisation constants on the Higgs fields to the one we introduce directly after the physical fields are defined. This may help in comparing different approaches in the literature.

To avoid clutter we use some abbreviations for the trigonometric functions. For example for an angle  $\theta$ ,  $\cos\theta$  will be abbreviated as  $c_\theta$ , *etc...* so that we will from now on use  $t_\beta$  for  $\tan\beta$ .

## 2 The Higgs sector of the MSSM at tree-level

### 2.1 The Higgs Potential

As known, see for instance [3], the MSSM requires two Higgs doublets  $H_1$  and  $H_2$  with opposite hypercharge. The Higgs potential in the MSSM is given by:

$$\begin{aligned} V &= m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_{12}^2 (H_1 \wedge H_2 + h.c.) \\ &+ \frac{1}{8} (g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2} |H_1^\dagger H_2|^2 \\ &\text{with } H_1 \wedge H_2 = H_1^a H_2^b \epsilon_{ab} \quad (\epsilon_{12} = -\epsilon_{21} = 1, \epsilon_{ii} = 0). \end{aligned} \quad (2.1)$$

The mass terms are all soft masses even if both  $m_1^2$  and  $m_2^2$  contain the SUSY preserving  $|\mu|^2$  term which originates from the F-terms.  $g, g'$  are, respectively, the  $SU(2)_W$  and  $U(1)_Y$  gauge couplings. Decomposing each Higgs doublet field  $H_{1,2}$  in terms of its components,

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} (v_1 + \phi_1^0 - i\varphi_1^0)/\sqrt{2} \\ -\phi_1^- \end{pmatrix}, \quad (2.2)$$

$$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ (v_2 + \phi_2^0 + i\varphi_2^0)/\sqrt{2} \end{pmatrix}, \quad (2.3)$$

the tree-level Higgs potential writes as

$$V = V_{const} + V_{linear} + V_{mass} + V_{cubic} + V_{quartic}, \quad (2.4)$$

where,

$$\begin{aligned}
V_{linear} &= \left( m_1^2 v_1 + m_{12}^2 v_2 + \frac{g^2 + g'^2}{8} (v_1^2 - v_2^2) v_1 \right) \phi_1^0 \\
&+ \left( m_2^2 v_2 + m_{12}^2 v_1 - \frac{g^2 + g'^2}{8} (v_1^2 - v_2^2) v_2 \right) \phi_2^0 \\
&\equiv T_{\phi_1^0} \phi_1^0 + T_{\phi_2^0} \phi_2^0,
\end{aligned} \tag{2.5}$$

and

$$\begin{aligned}
V_{mass} &= \frac{1}{2} \begin{pmatrix} \phi_1^0 & \phi_2^0 \end{pmatrix} \underbrace{\begin{pmatrix} m_1^2 + \frac{g^2 + g'^2}{8} (v_1^2 - v_2^2) & m_{12}^2 \\ m_{12}^2 & m_2^2 - \frac{g^2 + g'^2}{8} (v_1^2 - v_2^2) \end{pmatrix}}_{M_{\varphi^0}^2} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \\
&+ \frac{1}{2} \begin{pmatrix} \phi_1^0 & \phi_2^0 \end{pmatrix} \underbrace{\left( M_{\varphi^0}^2 + \frac{g^2 + g'^2}{4} \begin{pmatrix} v_1^2 & -v_1 v_2 \\ -v_1 v_2 & v_2^2 \end{pmatrix} \right)}_{M_{\phi^0}^2} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \\
&+ \begin{pmatrix} \phi_1^- & \phi_2^- \end{pmatrix} \underbrace{\left( M_{\varphi^0}^2 + \frac{g^2}{4} \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \right)}_{M_{\phi^\pm}^2} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}.
\end{aligned} \tag{2.6}$$

It is illuminating to express the mass matrices in terms of the tadpoles especially for the pseudoscalar states

$$\begin{aligned}
M_{\varphi^0}^2 &= \begin{pmatrix} \frac{T_{\phi_1^0}}{v_1} & 0 \\ 0 & \frac{T_{\phi_2^0}}{v_2} \end{pmatrix} - \frac{m_{12}^2}{v_1 v_2} N_{GP} \quad \text{with} \quad N_{GP} = \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}, \\
M_{\phi^c}^2 &= \begin{pmatrix} \frac{T_{\phi_1^0}}{v_1} & 0 \\ 0 & \frac{T_{\phi_2^0}}{v_2} \end{pmatrix} - \left( \frac{m_{12}^2}{v_1 v_2} - \frac{g^2}{4} \right) N_{GP}.
\end{aligned} \tag{2.7}$$

The requirement that  $v_1$  and  $v_2$  correspond to the true vacua is a requirement on the vanishing of the tadpoles. The tadpoles, by the way, are also a trade-off for  $m_1^2$  and  $m_2^2$ . Indeed note that expressing everything in terms of  $T_{\phi_{1,2}^0}$ , all explicit dependence on  $m_1^2$  and  $m_2^2$  has disappeared, even in the scalar (CP-even) sector. Note that once the tadpole condition has been imposed

$$T_{\phi_{1,2}^0} = 0, \tag{2.8}$$

we immediately find that in both the charged sector and pseudo-scalar sector, there is a Goldstone boson (*i.e.* a zero mass eigenvalue). This is immediate from the fact that

$$\det(N_{GP}) = 0. \tag{2.9}$$

The masses of the physical charged Higgs,  $M_{H^\pm}$  and the pseudoscalar Higgs,  $M_{A^0}$ , are then just set from *the invariant* obtained from

$$\text{Tr}(N_{GP}) = v_1^2 + v_2^2 = v^2, \tag{2.10}$$

which is another way of seeing that the combination  $v$  is a proper “observable”. Indeed after gauging we will find that the masses of the weak gauge bosons are

$$\begin{aligned}
M_{W^\pm}^2 &= \frac{1}{4} g^2 v^2, \\
M_{Z^0}^2 &= \frac{1}{4} (g^2 + g'^2) v^2.
\end{aligned} \tag{2.11}$$

Then

$$M_{A^0}^2 = \text{Tr}(M_{\phi^0}^2) = -m_{12}^2 \frac{v^2}{v_1 v_2} = m_1^2 + m_2^2, \quad (2.12)$$

$$M_{H^\pm}^2 = M_{A^0}^2 + M_{W^\pm}^2. \quad (2.13)$$

In Eq. (2.12), the first equality does show an implicit dependence on the ratio of vev ( $t_\beta$ ), but not through  $m_1^2 + m_2^2$ . The latter must be basis independent, as is the *combination*  $m_{12}^2/v_1 v_2$ . This is to keep in mind.

It is also interesting to note that for the scalar Higgses, there is a simple sum rule that does not involve any ratio of vev's. Indeed, taking the trace of  $M_{\phi^0}^2$  and call the two physical CP-even Higgses  $h^0$ , with mass  $M_{h^0}$ , and  $H^0$ , with mass  $M_{H^0}$ , that would be obtained after rotation, we get the sum rule

$$M_{h^0}^2 + M_{H^0}^2 = M_{A^0}^2 + M_{Z^0}^2. \quad (2.14)$$

$h^0$  will denote the lightest CP-even Higgs. Let us as a book-keeping device introduce the angle  $\beta$ . At the moment this is just to help have easy notations:

$$c_\beta = \frac{v_1}{v}, \quad s_\beta = \frac{v_2}{v} \quad \text{with } v = \sqrt{v_1^2 + v_2^2}. \quad (2.15)$$

The determinant of the scalar Higgses on the other hand gives

$$M_{h^0}^2 M_{H^0}^2 = M_{A^0}^2 M_{Z^0}^2 c_{2\beta}^2. \quad (2.16)$$

This shows that if we take  $M_{H^0}, M_{A^0}, M_{Z^0}$  as input parameters, we first derive  $M_{h^0}$  from Eq. (2.14), then  $c_{2\beta}^2$  from Eq. (2.16). In general with a set of input parameters  $M_{H^0}, M_{A^0}, M_{Z^0}$ ,  $c_{2\beta}^2 \leq 1$  is not guaranteed though. We could of course fix  $c_{2\beta}^2(t_\beta)$  and derive  $M_{H^0}$  and  $M_{h^0}$  which is what is usually done.

The soft SUSY breaking mass parameters  $m_{1,2,12}^2$  can be expressed in terms of the physical quantities,  $M_{A^0}, M_{Z^0}$  and  $c_\beta$  (as for example derived from Eqs. (2.14-2.16)):

$$m_1^2 = s_\beta^2 M_{A^0}^2 - \frac{1}{2} c_{2\beta}^2 M_{Z^0}^2, \quad (2.17)$$

$$m_{12}^2 = -\frac{1}{2} s_{2\beta} M_{A^0}^2, \quad (2.18)$$

$$m_2^2 = c_\beta^2 M_{A^0}^2 + \frac{1}{2} c_{2\beta}^2 M_{Z^0}^2. \quad (2.19)$$

## 2.2 Basis and rotations

So far the properties of the physical fields like their masses have been derived without reverting to a specific basis. The angle  $\beta$  defined in Eq. (2.15) was just a book-keeping device. Still, to go from the fields at the Lagrangian level to the physical fields one needs to perform a rotation. This should have no effect on physical observables. This naive observation is important especially when we move to one-loop. The rotations we will perform will get rid of field mixing. With the tadpole condition set to zero, it is clear that the pseudoscalar and charged scalars eigenstates are diagonalised through the same unitary matrix. At tree-level this is defined precisely through the same angle  $\beta$  as in Eq. (2.15),

$$N_{GP} = U(-\beta) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} U(\beta), \quad U(\beta) = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \quad U^\dagger(\beta) = U(-\beta). \quad (2.20)$$

Call  $\mathcal{T}_v$ , the tadpole matrix defined as

$$\mathcal{T}_v = \begin{pmatrix} \frac{T_{\phi_1^0}}{v_1} & 0 \\ 0 & \frac{T_{\phi_2^0}}{v_2} \end{pmatrix}. \quad (2.21)$$

The tadpole is, of course, set to zero. But we will leave this zero there in the notation as we will need this when we go to the one-loop counterterms. Then the mass matrices for the CP-even, CP-odd and charged scalars write

$$M_{\varphi^0}^2 = \mathcal{T}_v + M_{A^0}^2 N_{GP}, \quad (2.22)$$

$$M_{\phi^\pm}^2 = \mathcal{T}_v + (M_{A^0}^2 + M_{W^\pm}^2) N_{GP}, \quad (2.23)$$

$$M_{\phi^0}^2 = \mathcal{T}_v + M_{A^0}^2 N_{GP} + M_{Z^0}^2 U(\beta) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} U(-\beta). \quad (2.24)$$

The neutral Higgs is diagonalised through a rotation  $\alpha$  such that

$$\begin{aligned} U(\alpha) M_{\phi^0}^2 U(-\alpha) &= \begin{pmatrix} M_{H^0}^2 & 0 \\ 0 & M_{h^0}^2 \end{pmatrix} = U(\alpha) \mathcal{T}_v U(-\alpha) + \\ &M_{A^0}^2 U(\alpha - \beta) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} U(\beta - \alpha) + M_{Z^0}^2 U(\alpha + \beta) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} U(-(\alpha + \beta)). \end{aligned} \quad (2.25)$$

The diagonalisation procedure also produces other, sometimes useful, constraints and relations:

$$M_{H^0}^2 = M_{A^0}^2 s_{\alpha-\beta}^2 + M_{Z^0}^2 c_{\alpha+\beta}^2, \quad (2.26)$$

$$M_{h^0}^2 = M_{A^0}^2 c_{\alpha-\beta}^2 + M_{Z^0}^2 s_{\alpha+\beta}^2, \quad (2.27)$$

$$M_{A^0}^2 s_{2(\alpha-\beta)} = M_{Z^0}^2 c_{2(\alpha+\beta)}, \quad (2.28)$$

$$t_{2\alpha} = t_{2\beta} \frac{M_{A^0}^2 + M_{Z^0}^2}{M_{A^0}^2 - M_{Z^0}^2}. \quad (2.29)$$

Note that in the decoupling limit,  $M_{A^0} \gg M_{Z^0}$ , one has in effect decoupled one of the Higgs doublet, the other has the properties of the SM Higgs doublet. The decoupling parameter is also measured with the parameter  $c_{\beta-\alpha} \rightarrow M_{Z^0}^2/M_{A^0}^2$  for  $M_{A^0} \gg M_{Z^0}$ .

Therefore, the mass eigenstates in the Higgs sector are given by

$$\begin{aligned} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} &= U(\beta) \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}, \\ \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} &= U(\beta) \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \\ \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} &= U(\alpha) \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}. \end{aligned} \quad (2.30)$$

### 2.3 Counting parameters

Before we embark on the technicalities of renormalisation and the choice of judicious input parameters, it is best to review how we are going to proceed in general and how to make contact with the renormalisation of the SM. This will help clarify what are the fundamental parameters and which are the physical parameters that can be used for a legitimate renormalisation scheme. Moreover since some observables *belong* to the SM, like the  $W^\pm$ ,  $Z^0$  masses and the electromagnetic coupling constant  $e$  which are used as physical input parameters in the OS scheme, isolating these three parameters means that their renormalisation will proceed exactly as within the OS renormalisation of the SM, see [9] for details.

In the SM, the fundamental parameters at the Lagrangian level for the gauge sector are  $g$  and  $g'$ . The Higgs potential with the Higgs doublet  $\mathcal{H}$

$$\begin{aligned} V(\mathcal{H}) &= -\mu^2 \mathcal{H}^\dagger \mathcal{H} + \lambda (\mathcal{H}^\dagger \mathcal{H})^2 \quad \text{with} \\ |\langle 0 | \mathcal{H} | 0 \rangle|^2 &= \frac{v^2}{2} \neq 0, \end{aligned} \quad (2.31)$$

furnishes the following:  $\mu^2$  (the “Higgs mass”),  $\lambda$  (the Higgs self coupling) and  $v$  (the value of the vacuum expectation value). We thus have at Lagrangian level, 5 parameters between the Higgs sector and the gauge sector.  $\mu^2, \lambda, v$  are not all independent.  $v$ , the vacuum expectation value (vev), is defined as the minimum of the potential, this is equivalent to requiring no tadpoles. The no tadpole requirement amounts to no terms linear in the scalar Higgs. With the tadpole defined as  $T$ , we have at tree-level

$$T = v(\mu^2 - \lambda v^2) \rightarrow 0. \quad (2.32)$$

This requirement is to be carried to any loop level. Out of this constraint, the 5 physical parameters in the OS scheme are  $e, M_{W^\pm}, M_{Z^0}, M_{H^0}, T$ . At all orders one defines,  $c_W = M_{W^\pm}/M_{Z^0}$ . The latter is not an independent physical parameter. Therefore in the SM a one-to-one mapping between the physical set  $e, M_{W^\pm}, M_{Z^0}, M_{H^0}, T$  and the Lagrangian parameters  $g, g', v, \mu, \lambda$  is made.

In the MSSM, the Higgs sector furnishes  $m_1^2, m_2^2, m_{12}^2$  the Higgs doublets soft masses and  $v_1, v_2$  the vev of the Higgs doublets. The gauge sector is still governed by the  $U(1)_Y$  and  $SU(2)_W$  gauge couplings  $g, g'$ . The requirement of no tadpoles from both Higgs doublets, and hence any linear combination of them, is also a strong constraint. From these seven parameters in all, the physical parameters are usually split between the SM physical On-Shell parameters

$$e, M_{W^\pm}, M_{Z^0}, \quad (2.33)$$

which are a trade-off for  $g, g', v^2 = v_1^2 + v_2^2$  and the MSSM Higgs parameters

$$M_{A^0}, T_{\phi_1^0}, T_{\phi_2^0}; “t_\beta”, \quad (2.34)$$

which are a trade-off for  $m_1^2, m_2^2, m_{12}^2, v_2/v_1$ . At tree-level we can set  $t_\beta = v_2/v_1$  but this is, as yet, not directly related to an observable. While  $v$  can directly be expressed as a physical gauge boson mass, the ratio  $v_2/v_1$  within the Higgs sector does not have an immediate simple physical interpretation. Hence the difficulty with this Lagrangian parameter. One possibility is to trade it with the mass of one of the CP-even neutral Higgs through Eq. (2.16).

### 3 Non-linear gauge fixing

In SloopS we have generalised the usual 't Hooft linear gauge condition to a more general non-linear gauge that involves, thanks to the extra scalars in the Higgs sector, eight extra parameters  $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\omega}, \tilde{\kappa}, \tilde{\rho}, \tilde{\epsilon}, \tilde{\gamma})$ . Such gauges within the Standard Model had proved useful and powerful [10, 9] to check the correctness of the calculation. We have also exploited these gauges in the one-loop calculation of  $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \gamma\gamma, Z^0\gamma$  [11] and to corrections to the relic density in [12]. A 7-parameter non-linear gauge-fixing Lagrangian based on the one we introduce here is used in [13]. We can extend this non-linear gauge fixing so that the gauge-fixing function involves the sfermions also. We refrain, in this paper, from working through this generalisation.

We will take these gauge fixing terms to be *renormalised*. In particular the gauge functions involve the physical fields. Although this will not make all Green's functions finite, it is enough to make all  $S$ -matrix elements finite. The gauge-fixing writes

$$\mathcal{L}^{GF} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z} |F^Z|^2 - \frac{1}{2\xi_\gamma} |F^\gamma|^2, \quad (3.1)$$

where

$$\begin{aligned} F^+ &= (\partial_\mu - ie\tilde{\alpha}\gamma_\mu - ie\frac{c_W}{s_W}\tilde{\beta}Z_\mu)W^{\mu+} + i\xi_W\frac{e}{2s_W}(v + \tilde{\delta}h^0 + \tilde{\omega}H^0 + i\tilde{\rho}A^0 + i\tilde{\kappa}G^0)G^+, \\ F^Z &= \partial_\mu Z^\mu + \xi_Z\frac{e}{s_{2W}}(v + \tilde{\epsilon}h^0 + \tilde{\gamma}H^0)G^0, \\ F^\gamma &= \partial_\mu \gamma^\mu. \end{aligned} \quad (3.2)$$

The ghost Lagrangian  $\mathcal{L}^{Gh}$  is derived by requiring that the full effective Lagrangian,  $\mathcal{L}^Q$ , be invariant under the BRST transformation. As discussed in [9], this is a much more appropriate procedure than the usual Fadeev-Popov approach especially when dealing with the quantum symmetries of the generalised non-linear gauges we are using.  $\delta_{\text{BRS}}\mathcal{L}^Q = 0$  therefore implies  $\delta_{\text{BRS}}\mathcal{L}^{GF} = -\delta_{\text{BRS}}\mathcal{L}^{Gh}$ .

It is very useful to also introduce the auxiliary  $B$ -field formulation of the gauge-fixing Lagrangian  $\mathcal{L}^{GF}$ , especially from the perspective of deriving some Ward identities. The gauge fixing can then be expressed as

$$\mathcal{L}^{GF} = \xi_W B^+ B^- + \frac{\xi_Z}{2} |B^Z|^2 + \frac{\xi_\gamma}{2} |B^\gamma|^2 + B^- F^+ + B^+ F^- + B^Z F^Z + B^\gamma F^\gamma. \quad (3.3)$$

From the equations of motion for the  $B$ -fields we recover the usual  $\mathcal{L}^{GF}$  together with the condition  $B^i = -\frac{F^i}{\xi_i}$  ( $\xi_i = \{\xi_W, \xi_Z, \xi_\gamma\}$ ). The anti-ghost,  $\bar{c}^i$ , is defined from the gauge fixing functions, we write

$$\delta_{\text{BRS}}\bar{c}^i = B^i. \quad (3.4)$$

Then the ghost Lagrangian writes as

$$\mathcal{L}^{Gh} = -(\bar{c}^+ \delta_{\text{BRS}} F^+ + \bar{c}^- \delta_{\text{BRS}} F^- + \bar{c}^Z \delta_{\text{BRS}} F^Z + \bar{c}^\gamma \delta_{\text{BRS}} F^\gamma) \delta_{\text{BRS}} \tilde{\mathcal{L}}^{Gh}. \quad (3.5)$$

The Fadeev-Popov prescription is therefore readily recovered,  $\mathcal{L}^{FP}$ , but only up to an overall function,  $\delta_{\text{BRS}}\tilde{\mathcal{L}}^{Gh}$ , which is BRST invariant. The latter is set to zero for one-loop calculations. Our code `SloopS` implements this prescription automatically leading to the automatic generation of the whole set of Feynman rules for the ghost sector.

For all applications we set the Feynman parameters  $\xi_{W,Z,\gamma}$  to one. This allows one to use the minimum set of libraries for the tensor reduction. Indeed,  $\xi_{W,Z,\gamma} \neq 1$  can generate high rank tensor loop functions, that would take much time to reduce to the set of scalar functions.

It is important to stress, once more, that since we do not seek to have all Green's functions finite but only the S-matrix elements, we take the gauge fixing Lagrangian as being renormalised.

Judicious choices of the the non-linear gauge parameters can lead to simplifications like the vanishing of certain vertices. For example, with  $\tilde{\alpha} = 1$ , the  $W^+{}^\mu G^- \gamma_\mu$  vertex cancels. More examples can be found in Appendix A for the vanishing of some ghost couplings to Higgses.

## 4 Renormalisation

Our renormalisation procedure is within the spirit of the on-shell scheme borrowing as much as possible from the programme carried strictly within the Standard Model in [9]. For the gauge sector and the fermion sector, beside the electromagnetic coupling which we fix from the Thomson limit, we take therefore the same set of physical input parameters, namely the masses of the  $W^\pm$  and  $Z^0$  together with the masses of all the standard model fermions. To define the Higgs sector parameters, the set of Eq. (2.34) looks most appropriate were it not for the ill defined  $t_\beta$ . Indeed, the mass of the pseudoscalar  $M_{A^0}$  within the MSSM with CP conservation is a physical parameter. As within the Standard Model, we also take the tadpole. For  $t_\beta$  the aim of this paper is to review, propose and compare different schemes. Renormalisation of these parameters would then lead to finite S-matrix elements. For the mass eigenstates and thus a proper identification of the physical particles that appear as external legs in our processes, field renormalisation is needed. S-matrix elements obtained from these rescaled Green's functions will lead to external legs with unit residue and will avoid mixing. Therefore one also needs wave function renormalisation of the fields. Especially for the unphysical sector of the theory, the precise choice of the fields redefinition is not essential if one is only interested in S-matrix elements of physical processes. It has to be stressed that one can do without this if one is willing to include loop corrections on the external



legs. In the MSSM and in the Higgs sector in particular mixing effects, especially at one-loop, are a nuisance that has introduced some confusion especially in defining  $t_\beta$  with the help of wave-function renormalisation constants or equivalently from two-point function describing particle mixing. For the Higgs sector one needs to be wary about mixing of the Goldstones with CP-odd Higgs or almost equivalently between the  $Z^0$  and the CP-odd Higgs or the  $W^\pm$  and the charged Higgs. These two-point functions are related through gauge invariance and impose strong constraints on the wave function renormalisation constants. We will derive Ward-Slavnov-Taylor identities relating these two-point functions, and hence their associated counterterms, before imposing any ad-hoc condition.

## 4.1 Shifts in mass parameters and gauge couplings

All fields and parameters introduced so far are considered as bare parameters with the exception of the gauge fixing Lagrangian which we choose to write in terms of *renormalised fields*. Care should then be exercised when we split the tree-level contributions and the counterterms. Shifts are then introduced for the Lagrangian parameters and the fields with the notation that a bare quantity is labeled as  $X_0$ . It will split in terms of renormalised quantities  $X$  and counterterms  $\delta X$

$$g_0 = g + \delta g, \quad g'_0 \rightarrow g' + \delta g', \quad (4.1)$$

$$m_{i0}^2 = m_i^2 + \delta m_i^2 \quad \text{for } i = 1, 2, \quad m_{120}^2 = m_{12}^2 + \delta m_{12}^2, \quad (4.2)$$

$$v_{i0} = v_i - \delta v_i \quad \text{for } i = 1, 2 \quad \text{hence} \quad \frac{\delta t_\beta}{t_\beta} = \frac{\delta v_1}{v_1} - \frac{\delta v_2}{v_2}. \quad (4.3)$$

In our approach the angles defining the rotation matrices,  $\beta$  and  $\alpha$  in Eq. (2.30) are defined as renormalised quantities. For example the relation between the Goldstone boson/pseudoscalar Higgs boson and the fields  $\varphi_{1,2}^0$  is maintained at all orders. Indeed,

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = U(\beta) \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}_0 \quad \text{implies also} \quad \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = U(\beta) \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}. \quad (4.4)$$

Since in our approach we will always perform a field renormalisation there is no need in inducing more shifts from  $U(\alpha, \beta)$ . Therefore  $U(\alpha, \beta)$  is taken as renormalised. For example, if we perform a field renormalisation in the  $\varphi_{1,2}^0$  basis

$$\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}_0 = Z_{\varphi^0} \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix} = \begin{pmatrix} Z_{\varphi_1^0}^{1/2} & Z_{\varphi_1^0 \varphi_2^0}^{1/2} \\ Z_{\varphi_2^0 \varphi_1^0}^{1/2} & Z_{\varphi_2^0}^{1/2} \end{pmatrix} \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}, \quad (4.5)$$

this will imply

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = U(\beta) Z_{\varphi^0} U(-\beta) \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = Z_P \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} Z_{G^0 G^0}^{1/2} & Z_{G^0 A^0}^{1/2} \\ Z_{A^0 G^0}^{1/2} & Z_{A^0 A^0}^{1/2} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}. \quad (4.6)$$

For the field renormalisation we can perform this either at the level of the  $\varphi_i^0$ , *i.e.* before any rotation on the field in the Lagrangian is made, through  $Z_{\varphi^0}$  as is done in [14, 15, 13] or in a much efficient way directly in the basis  $G^0 A^0$  since the latter are directly related to our renormalisation conditions on the physical fields as we will see later. For instance, there is no need for  $Z_{G^0 G^0}$  in our approach since we will not be dealing with Goldstone bosons in the external legs.

## 4.2 Tadpole terms

We start with the terms linear in the Higgs fields which will lead to renormalisation of the tadpoles. With the tree-level condition on the tadpoles  $T_{\phi_1^0} = T_{\phi_2^0} = 0$ , field normalisation if it were performed does not contribute, we therefore have

$$V_{linear}|_0 = (\delta T_{\phi_1^0} \phi_1^0 + \delta T_{\phi_2^0} \phi_2^0), \quad (4.7)$$

with

$$\begin{aligned} \frac{\delta T_{\phi_1^0}}{v_1} &= \frac{M_{Z^0}^2}{2} c_{2\beta} \frac{\delta g^2 + \delta g'^2}{g^2 + g'^2} + \delta m_1^2 + t_\beta \delta m_{12}^2 \\ &- \left( m_1^2 + \frac{M_{Z^0}^2}{2} c_{2\beta} + M_{Z^0}^2 c_\beta^2 \right) \frac{\delta v_1}{v_1} + \left( -m_{12}^2 + \frac{M_{Z^0}^2}{2} s_{2\beta} \right) t_\beta \frac{\delta v_2}{v_2}, \end{aligned} \quad (4.8)$$

$$\frac{\delta T_{\phi_2^0}}{v_2} = \frac{\delta T_{\phi_1^0}}{v_1} (v_1 \leftrightarrow v_2, m_1 \leftrightarrow m_2). \quad (4.9)$$

The minimum condition requires the one-loop tadpole contribution generated by one-loop diagrams,  $T_{\phi_i^0}^{\text{loop}}$  is cancelled by the tadpole counterterm. This imposes

$$\delta T_{\phi_i^0} = -T_{\phi_i^0}^{\text{loop}}. \quad (4.10)$$

$T_{\phi_i^0}^{\text{loop}}$  is calculated from the one-loop tadpole amplitude for  $H^0$ ,  $T_{H^0}^{\text{loop}}$  and  $h^0$ ,  $T_{h^0}^{\text{loop}}$  by simply moving to the physical basis

$$\begin{pmatrix} T_{\phi_1^0}^{\text{loop}} \\ T_{\phi_2^0}^{\text{loop}} \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} T_{H^0}^{\text{loop}} \\ T_{h^0}^{\text{loop}} \end{pmatrix}. \quad (4.11)$$

### 4.3 Mass counterterms in the Higgs sector

We now move to the mass counterterms induced by shifts in the Lagrangian parameters. We need to consider all terms bi-linear in the fields. From the bare matrices  $M_{\varphi^0}^2$ ,  $M_{\phi^\pm}^2$  and  $M_{\phi^0}^2$  (Eqs. (2.6), (2.22) - (2.24)), we find the corresponding counterterms in matrix form in the basis  $\varphi_{1,2}^0$ ,  $\phi_{1,2}^0$  and  $\phi_{1,2}^\pm$

$$\begin{aligned} \delta M_{\varphi^0}^2 &= \begin{pmatrix} \delta m_1^2 + \frac{1}{2} c_{2\beta} \delta M_{Z^0}^2 - \frac{M_{Z^0}^2}{2} s_{2\beta}^2 \frac{\delta t_\beta}{t_\beta} & \delta m_{12}^2 \\ \delta m_{12}^2 & \delta m_2^2 - \frac{1}{2} c_{2\beta} \delta M_{Z^0}^2 + \frac{M_{Z^0}^2}{2} s_{2\beta}^2 \frac{\delta t_\beta}{t_\beta} \end{pmatrix} \\ \delta M_{\phi^\pm}^2 &= \begin{pmatrix} \delta m_1^2 + \frac{1}{2} c_{2\beta} \delta M_{Z^0}^2 + s_\beta^2 \delta M_{W^\pm}^2 - \frac{M_{Z^0}^2}{2} s_{2\beta}^2 s_W^2 \frac{\delta t_\beta}{t_\beta} & \delta m_{12}^2 - \frac{1}{2} s_{2\beta} \delta M_{W^\pm}^2 - \frac{M_{W^\pm}^2}{4} s_{4\beta} \frac{\delta t_\beta}{t_\beta} \\ \delta m_{12}^2 - \frac{1}{2} s_{2\beta} \delta M_{W^\pm}^2 - \frac{M_{W^\pm}^2}{4} s_{4\beta} \frac{\delta t_\beta}{t_\beta} & \delta m_2^2 - \frac{1}{2} c_{2\beta} \delta M_{Z^0}^2 + c_\beta^2 \delta M_{W^\pm}^2 + \frac{M_{Z^0}^2}{2} s_{2\beta}^2 s_W^2 \frac{\delta t_\beta}{t_\beta} \end{pmatrix} \\ \delta M_{\phi^0}^2 &= \begin{pmatrix} \delta m_1^2 + \frac{1}{2} (4c_\beta^2 - 1) \delta M_{Z^0}^2 - M_{Z^0}^2 s_{2\beta}^2 \frac{\delta t_\beta}{t_\beta} & \delta m_{12}^2 - \frac{1}{2} s_{2\beta} \delta M_{Z^0}^2 - \frac{M_{Z^0}^2}{4} s_{4\beta} \frac{\delta t_\beta}{t_\beta} \\ \delta m_{12}^2 - \frac{1}{2} s_{2\beta} \delta M_{Z^0}^2 - \frac{M_{Z^0}^2}{4} s_{4\beta} \frac{\delta t_\beta}{t_\beta} & \delta m_2^2 + \frac{1}{2} (4s_\beta^2 - 1) \delta M_{Z^0}^2 + M_{Z^0}^2 s_{2\beta}^2 \frac{\delta t_\beta}{t_\beta} \end{pmatrix} \end{aligned}$$

It is then straightforward to move to the physical fields through the rotation matrices  $U(\alpha)$  and  $U(\beta)$ , to find the mass counterterms  $\delta M_{A^0}^2$ ,  $\delta M_{H^\pm}^2$ ,  $\delta M_{h^0}^2$ ,  $\delta M_{H^0}^2$  for, respectively, the pseudoscalar Higgs,  $A^0$ , the charged Higgs  $H^\pm$ , and the two CP-even Higgses  $h^0$ ,  $H^0$ . A mass mixing between these two Higgses,  $\delta M_{H^0 h^0}^2$  is also induced

$$\begin{aligned}
\delta M_{A^0}^2 &= s_\beta^2 \delta m_1^2 + c_\beta^2 \delta m_2^2 - s_{2\beta} \delta m_{12}^2 - \frac{1}{2} c_{2\beta}^2 \delta M_{Z^0}^2 + \frac{M_{Z^0}^2}{2} s_{2\beta}^2 c_{2\beta} \frac{\delta t_\beta}{t_\beta}, \\
\delta M_{H^\pm}^2 &= \delta M_{A^0}^2 + \delta M_{W^\pm}^2, \\
\delta M_{H^0}^2 &= c_\alpha^2 \delta m_1^2 + s_\alpha^2 \delta m_2^2 + s_{2\alpha} \delta m_{12}^2 \\
&\quad + \frac{1}{2} \left( 4(c_\alpha^2 c_\beta^2 + s_\alpha^2 s_\beta^2 - c_\alpha s_\alpha c_\beta s_\beta) - 1 \right) \delta M_{Z^0}^2 - \frac{M_{Z^0}^2}{2} s_{2\beta} \left( 2c_{2\alpha} s_{2\beta} + s_{2\alpha} c_{2\beta} \right) \frac{\delta t_\beta}{t_\beta}, \\
\delta M_{h^0}^2 &= s_\alpha^2 \delta m_1^2 + c_\alpha^2 \delta m_2^2 - s_{2\alpha} \delta m_{12}^2 \\
&\quad + \frac{1}{2} \left( 4(c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2 + c_\alpha s_\alpha c_\beta s_\beta) - 1 \right) \delta M_{Z^0}^2 + \frac{M_{Z^0}^2}{2} s_{2\beta} \left( 2c_{2\alpha} s_{2\beta} + s_{2\alpha} c_{2\beta} \right) \frac{\delta t_\beta}{t_\beta}, \\
\delta M_{H^0 h^0}^2 &= c_{2\alpha} \delta m_{12}^2 + \frac{1}{2} s_{2\alpha} (\delta m_2^2 - \delta m_1^2) \\
&\quad - \frac{1}{2} \left( 2s_{2\alpha} c_{2\beta} + s_{2\beta} c_{2\alpha} \right) \delta M_{Z^0}^2 + \frac{M_{Z^0}^2}{2} s_{2\beta} \left( 2s_{2\alpha} s_{2\beta} - c_{2\alpha} c_{2\beta} \right) \frac{\delta t_\beta}{t_\beta}. \tag{4.12}
\end{aligned}$$

A mass term seems to be induced for the Goldstone bosons as well as a mixing between the Goldstones and the corresponding CP-odd Higgs

$$\delta M_{G^0}^2 = c_\beta^2 \delta m_1^2 + s_\beta^2 \delta m_2^2 + s_{2\beta} \delta m_{12}^2 + \frac{1}{2} c_{2\beta}^2 \delta M_{Z^0}^2 - \frac{1}{2} M_{Z^0}^2 s_{2\beta}^2 c_{2\beta} \frac{\delta t_\beta}{t_\beta}, \tag{4.13}$$

$$\delta M_{G^\pm}^2 = \delta M_{G^0}^2, \tag{4.14}$$

$$\delta M_{G^0 A^0}^2 = c_{2\beta} \delta m_{12}^2 + c_\beta s_\beta (\delta m_2^2 - \delta m_1^2) - \frac{1}{2} c_{2\beta} s_{2\beta} \delta M_{Z^0}^2 + M_{Z^0}^2 s_{2\beta}^2 c_\beta s_\beta \frac{\delta t_\beta}{t_\beta}, \tag{4.15}$$

$$\delta M_{G^\pm H^\pm}^2 = \delta M_{G^0 A^0}^2 - M_{W^\pm}^2 c_\beta s_\beta \frac{\delta t_\beta}{t_\beta}. \tag{4.16}$$

It is much more transparent to re-express these mass counterterms by trading-off  $\delta m_{1,2}$  and  $\delta m_{12}$  with our input parameters  $\delta T_{\phi_{1,2}^0}, \delta M_{A^0}^2, \delta t_\beta$  through

$$\begin{aligned}
\delta m_1^2 &= c_\beta^2 (s_\beta^2 + 1) \frac{\delta T_{\phi_1^0}}{v_1} - c_\beta^2 s_\beta^2 \frac{\delta T_{\phi_2^0}}{v_2} + s_\beta^2 \delta M_{A^0}^2 - \frac{1}{2} c_{2\beta} \delta M_{Z^0}^2 + \frac{1}{2} s_{2\beta}^2 (M_{A^0}^2 + M_{Z^0}^2) \frac{\delta t_\beta}{t_\beta}, \\
\delta m_{12}^2 &= \frac{1}{2} s_{2\beta} \left( s_\beta^2 \frac{\delta T_{\phi_1^0}}{v_1} + c_\beta^2 \frac{\delta T_{\phi_2^0}}{v_2} - \delta M_{A^0}^2 - c_{2\beta} M_{A^0}^2 \frac{\delta t_\beta}{t_\beta} \right), \\
\delta m_2^2 &= -c_\beta^2 s_\beta^2 \frac{\delta T_{\phi_1^0}}{v_1} + s_\beta^2 (c_\beta^2 + 1) \frac{\delta T_{\phi_2^0}}{v_2} + c_\beta^2 \delta M_{A^0}^2 + \frac{1}{2} c_{2\beta} \delta M_{Z^0}^2 - \frac{1}{2} s_{2\beta}^2 (M_{A^0}^2 + M_{Z^0}^2) \frac{\delta t_\beta}{t_\beta}.
\end{aligned} \tag{4.17}$$

In terms of  $\delta T_{\phi_{1,2}^0}, \delta M_{A^0}^2, \delta t_\beta$ , the mass counterterms of Eq. (4.12) and Eq. (4.16) write

$$\begin{aligned}
\delta M_{G^0}^2 &= \delta M_{G^\pm}^2 = \frac{1}{v}(c_{\alpha-\beta}\delta T_{H^0} - s_{\alpha-\beta}\delta T_{h^0}), \\
\delta M_{G^0 A^0}^2 &= \frac{1}{v}(s_{\alpha-\beta}\delta T_{H^0} + c_{\alpha-\beta}\delta T_{h^0}) - s_{2\beta}\frac{M_{A^0}^2}{2}\frac{\delta t_\beta}{t_\beta}, \\
\delta M_{G^\pm H^\pm}^2 &= \frac{1}{v}(s_{\alpha-\beta}\delta T_{H^0} + c_{\alpha-\beta}\delta T_{h^0}) - s_{2\beta}\frac{M_{H^\pm}^2}{2}\frac{\delta t_\beta}{t_\beta}, \\
\delta M_{H^\pm}^2 &= \delta M_{A^0}^2 + \delta M_{W^\pm}^2, \\
\delta M_{h^0}^2 &= -\frac{1}{v}\left(c_{\alpha-\beta}s_{\alpha-\beta}^2\delta T_{H^0} + s_{\alpha-\beta}(1 + c_{\alpha-\beta}^2)\delta T_{h^0}\right) + c_{\alpha-\beta}^2\delta M_{A^0}^2 + s_{\alpha+\beta}^2\delta M_{Z^0}^2 \\
&\quad + s_{2\beta}s_{2(\alpha+\beta)}M_{Z^0}^2\frac{\delta t_\beta}{t_\beta}, \\
\delta M_{H^0}^2 &= \frac{1}{v}\left(c_{\alpha-\beta}(1 + s_{\alpha-\beta}^2)\delta T_{H^0} + s_{\alpha-\beta}c_{\alpha-\beta}^2\delta T_{h^0}\right) + s_{\alpha-\beta}^2\delta M_{A^0}^2 + c_{\alpha+\beta}^2\delta M_{Z^0}^2 \\
&\quad - s_{2\beta}s_{2(\alpha+\beta)}M_{Z^0}^2\frac{\delta t_\beta}{t_\beta}, \\
\delta M_{H^0 h^0} &= -\frac{1}{v}s_{\alpha-\beta}^3\delta T_{H^0} + \frac{1}{v}c_{\alpha-\beta}^3\delta T_{h^0} + \frac{1}{2}s_{2(\alpha-\beta)}\delta M_{A^0}^2 - \frac{1}{2}s_{2(\alpha+\beta)}\delta M_{Z^0}^2 - \frac{s_{2\beta}}{2}\left(M_{A^0}^2 c_{2(\alpha-\beta)}\right. \\
&\quad \left.+ M_{Z^0}^2 c_{2(\alpha+\beta)}\right)\frac{\delta t_\beta}{t_\beta}. \tag{4.18}
\end{aligned}$$

It is very satisfying to see that  $\delta M_{G^0}^2 = \delta M_{G^\pm}^2$  is accounted for totally by the tadpole counterterms.

#### 4.4 Field renormalisation

We can now introduce field renormalisation at the level of the physical fields without the need to first go through field renormalisation in the basis  $\phi_{1,2}^0, \varphi_{1,2}^0, \phi_{1,2}^\pm$ . In most generality we can write, as in Eq. (4.6)

$$\begin{aligned}
\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 &= Z_P \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \equiv \begin{pmatrix} Z_{G^0}^{1/2} & Z_{G^0 A^0}^{1/2} \\ Z_{A^0 G^0}^{1/2} & Z_{A^0}^{1/2} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \\
\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}_0 &= Z_C \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \equiv \begin{pmatrix} Z_{G^\pm}^{1/2} & Z_{G^\pm H^\pm}^{1/2} \\ Z_{H^\pm G^\pm}^{1/2} & Z_{H^\pm}^{1/2} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}, \\
\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 &= Z_S \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} \equiv \begin{pmatrix} Z_{H^0}^{1/2} & Z_{H^0 h^0}^{1/2} \\ Z_{h^0 H^0}^{1/2} & Z_{h^0}^{1/2} \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}. \tag{4.19}
\end{aligned}$$

It is always possible to move basis through Eq. (4.6). Field renormalisation will help get rid of mixing between physical fields when these are on-shell and set the residue to 1.

## 4.5 Self-energies in the Higgs sector

Collecting the contribution of all the counterterms, including wave function renormalisation, the renormalised self-energies write as

$$\begin{cases} \hat{\Sigma}_{G^0 G^0}(q^2) = \Sigma_{G^0 G^0}(q^2) + \delta M_{G^0}^2 - q^2 \delta Z_{G^0} \\ \hat{\Sigma}_{G^0 A^0}(q^2) = \Sigma_{G^0 A^0}(q^2) + \delta M_{G^0 A^0}^2 - \frac{1}{2} q^2 \delta Z_{G^0 A^0} - \frac{1}{2} (q^2 - M_{A^0}^2) \delta Z_{A^0 G^0} \\ \hat{\Sigma}_{A^0 A^0}(q^2) = \Sigma_{A^0 A^0}(q^2) + \delta M_{A^0}^2 - (q^2 - M_{A^0}^2) \delta Z_{A^0} \\ \hat{\Sigma}_{G^\pm G^\pm}(q^2) = \Sigma_{G^\pm G^\pm}(q^2) + \delta M_{G^\pm}^2 - q^2 \delta Z_{G^\pm} \\ \hat{\Sigma}_{G^\pm H^\pm}(q^2) = \Sigma_{G^\pm H^\pm}(q^2) + \delta M_{G^\pm H^\pm}^2 - \frac{1}{2} q^2 \delta Z_{G^\pm H^\pm} - \frac{1}{2} (q^2 - M_{H^\pm}^2) \delta Z_{H^\pm G^\pm} \\ \hat{\Sigma}_{H^\pm H^\pm}(q^2) = \Sigma_{H^\pm H^\pm}(q^2) + \delta M_{H^\pm}^2 - (q^2 - M_{H^\pm}^2) \delta Z_{H^\pm} \\ \hat{\Sigma}_{H^0 H^0}(q^2) = \Sigma_{H^0 H^0}(q^2) + \delta M_{H^0}^2 - (q^2 - M_{H^0}^2) \delta Z_{H^0} \\ \hat{\Sigma}_{H^0 h^0}(q^2) = \Sigma_{H^0 h^0}(q^2) + \delta M_{H^0 h^0}^2 - \frac{1}{2} (q^2 - M_{H^0}^2) \delta Z_{H^0 h^0} - \frac{1}{2} (q^2 - M_{h^0}^2) \delta Z_{h^0 H^0} \\ \hat{\Sigma}_{h^0 h^0}(q^2) = \Sigma_{h^0 h^0}(q^2) + \delta M_{h^0}^2 - (q^2 - M_{h^0}^2) \delta Z_{h^0} \end{cases}$$

Note that as we stressed all along, since we are only interested in having finite  $S$ -matrix transitions and not finite Green's functions there is no need trying to make all two-point functions finite. For instance the diagonal Goldstone self-energies  $\hat{\Sigma}_{G^0 G^0}(q^2)$  and  $\hat{\Sigma}_{G^\pm G^\pm}(q^2)$  do not need any field renormalisation. Therefore we can set for example  $\delta Z_{G^0} = \delta Z_{G^\pm} = 0$  for simplicity.  $\delta Z_{A^0 G^0}$  is also not needed as it is only involved in the transition of Goldstone boson to the pseudo-scalar Higgs.

## 4.6 $A^0 Z^0$ and $H^\pm W^\pm$ transitions

The (massive) gauge bosons and the pseudo-scalar mix. This originates from the same part of the gauge Lagrangian where the gauge bosons, at tree-level, mix with the Goldstone bosons as in the Standard Model, see for example [9]. The latter is eliminated through the usual 't Hooft gauge fixing. To wit, from

$$\begin{aligned} \mathcal{L}_0^{GV} &= \frac{g}{2} i (v_1 \partial^\mu \phi_1^- + v_2 \partial^\mu \phi_2^-) W_\mu^+ + h.c. \\ &- \frac{g}{2c_W} (v_1 \partial^\mu \varphi_1^0 + v_2 \partial^\mu \varphi_2^0) Z_\mu^0|_0, \end{aligned} \quad (4.20)$$

we end up with

$$\begin{aligned} \mathcal{L}_0^{GV} = \mathcal{L}^{GV} &+ \frac{1}{2} \left( \delta Z_{G^\pm} + \delta Z_{W^\pm} + \frac{\delta M_{W^\pm}^2}{M_{W^\pm}^2} \right) (i M_{W^\pm} \partial^\mu G^- W_\mu^+ + h.c.) \\ &- \frac{1}{2} \left( \delta Z_{G^0} + \delta Z_{Z^0} + \frac{\delta M_{Z^0}^2}{M_{Z^0}^2} \right) M_{Z^0} \partial^\mu G^0 Z_\mu^0 \\ &- \frac{1}{2} \delta Z_{Z^0 \gamma} M_{Z^0} \partial^\mu G^0 \gamma_\mu \\ &+ \frac{1}{2} \left( \delta Z_{G^\pm H^\pm} + s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right) (i M_{W^\pm} \partial^\mu H^- W_\mu^+ + h.c.) \\ &- \frac{1}{2} \left( \delta Z_{G^0 A^0} + s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right) M_{Z^0} \partial^\mu A^0 Z_\mu^0. \end{aligned} \quad (4.21)$$

For the sake of completeness, we have also kept in Eq. (4.21) the wave-function renormalisation constants of the gauge bosons, namely  $\delta Z_{W^\pm}$ ,  $\delta Z_{Z^0}$  and  $\delta Z_{Z^0 \gamma}$  (for the  $Z^0 \rightarrow \gamma$  transition), see [9]. The conditions on the latter are the same as in the Standard Model, details are found in [9]. The novelty however is that now we have  $A^0 - Z^0$  and  $H^\pm - W^\pm$  transitions whose self-energies

write:

$$\hat{\Sigma}_{A^0 Z^0}(q^2) = \Sigma_{A^0 Z^0}(q^2) + \frac{M_{Z^0}}{2} \left( \delta Z_{G^0 A^0} + s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right), \quad (4.22)$$

$$\hat{\Sigma}_{H^\pm W^\pm}(q^2) = \Sigma_{H^\pm W^\pm}(q^2) + \frac{M_{W^\pm}}{2} \left( \delta Z_{G^\pm H^\pm} + s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right). \quad (4.23)$$

Note that apart from  $\delta t_\beta$  the same counterterm  $\delta Z_{G^0 A^0}$  appears in the  $G^0 A^0$  transition. In fact there is a Ward identity relating these two transitions. Contrary to what one might see in some papers [16, 17, 18], the relation is much more complicated for  $q^2 \neq M_{A^0}^2$  and gets more subtle in the case of the non-linear gauge. This identity is very important especially that in many approaches the transition has been used as a *definition* for  $\delta t_\beta$ . The identity can be most easily derived by considering the BRST transformation on the (“ghost”) operator  $\langle 0 | \bar{c}^Z(x) A^0(y) | 0 \rangle = 0$ . Full details are given in Appendix A. We have the constraint

$$\begin{aligned} q^2 \hat{\Sigma}_{A^0 Z^0}(q^2) + M_{Z^0} \hat{\Sigma}_{A^0 G^0}(q^2) &= (q^2 - M_{Z^0}^2) \frac{1}{(4\pi)^2} \frac{e^2 M_{Z^0}}{s_{2W}^2} s_{2\beta} \mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) \\ &+ \frac{M_{Z^0}}{2} (q^2 - M_{A^0}^2) \left( \frac{1}{(4\pi)^2} \frac{2e^2}{s_{2W}^2} \mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) + s_{2\beta} \frac{\delta t_\beta}{t_\beta} - \delta Z_{A^0 G^0} \right). \end{aligned} \quad (4.24)$$

$\mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2)$  and  $\mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2)$  are functions defined in Appendix A. They vanish in the linear gauge with  $\tilde{\epsilon} = \tilde{\gamma} = 0$ . The constraint shows that even in the linear gauge  $q^2 \hat{\Sigma}_{A^0 Z^0}(q^2) + M_{Z^0} \hat{\Sigma}_{A^0 G^0}(q^2)$  is zero only for  $q^2 = M_{A^0}^2$  and not for *any*  $q^2$ . We will get back to the exploitation of this constraint later. A similar constraint relates also  $\hat{\Sigma}_{H^\pm W^\pm}(q^2)$  and  $\hat{\Sigma}_{G^\pm H^\pm}(q^2)$

$$\begin{aligned} q^2 \hat{\Sigma}_{H^\pm W^\pm}(q^2) + M_{W^\pm} \hat{\Sigma}_{H^\pm G^\pm}(q^2) &= (q^2 - M_{W^\pm}^2) \frac{1}{(4\pi)^2} \frac{e^2 M_{W^\pm}}{s_{2W}^2} \mathcal{G}_{HW}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2) \\ &+ \frac{M_{W^\pm}}{2} (q^2 - M_{H^\pm}^2) \left( \frac{1}{(4\pi)^2} \frac{2e^2}{s_{2W}^2} \mathcal{G}_{cc}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2) + s_{2\beta} \frac{\delta t_\beta}{t_\beta} - \delta Z_{H^\pm G^\pm} \right). \end{aligned}$$

$\mathcal{G}_{HW}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2)$  and  $\mathcal{G}_{cc}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2)$  are defined in Eq. (A.26), see Appendix A.

## 4.7 Renormalisation conditions

### 4.7.1 Pole masses, residues and mixing

Masses are defined as pole masses from the propagator. Moreover this propagator must have residue 1 at the pole mass. In the case of particle mixing, the mixing must vanish at the pole mass of any physical particle, *i.e.* at the pole mass. In general in the case of mixing this requires solving a system of an inverse propagator matrix with solutions given by the pole masses. For a 2-particle mixing one has to deal with the determinant of a  $2 \times 2$  matrix which is a quadratic form in the self-energies whose solutions are the corrected masses. The equation reads

$$\left[ \left( q^2 - M_{h^0, \text{tree}}^2 - \hat{\Sigma}_{h^0 h^0}(q^2) \right) \left( q^2 - M_{H^0, \text{tree}}^2 - \hat{\Sigma}_{H^0 H^0}(q^2) \right) - \left( \hat{\Sigma}_{h^0 H^0}(q^2) \right)^2 \right] = 0. \quad (4.25)$$

$M_{h^0, \text{tree}}$  refers to the tree-level mass. This equations simplifies considerably at one-loop since one only has to keep the linear term, or first order in the loop expansion, in the equation. In principle the argument that appears in the self-energy two-point functions is the pole mass which might get a correction from its value at tree-level. To get the corrections one can proceed through iteration, starting from the tree-level masses as argument of the two-point function. Higher order terms in the expansion will appear as higher orders in the loop expansion and we do not count them as being part of the one-loop correction. A genuine one-loop results for the pole mass,  $M_{i, 1\text{loop}}$ , starting

from a tree-level mass  $M_{i,\text{tree}}$  with  $\hat{\Sigma}_{ii}(q^2)$  the diagonal renormalised self-energy is therefore the solution of

$$q^2 - M_{i,\text{tree}}^2 - \text{Re}\hat{\Sigma}_{ii}(q^2) = 0 \quad \text{at} \quad q^2 = M_{i,\text{1loop}}^2, \quad (4.26)$$

which in the one-loop approximation means

$$M_{i,\text{1loop}}^2 = M_{i,\text{tree}}^2 + \text{Re}\hat{\Sigma}_{ii}(M_{i,\text{tree}}^2) = M_{i,\text{tree}}^2 + \delta M_{ii}^2 + \text{Re}\Sigma_{ii}(M_{i,\text{tree}}^2). \quad (4.27)$$

The latter condition will constrain the Lagrangian parameters with  $\delta M_{ii}^2$  a gauge invariant quantity. Likewise, at one-loop, the requirement of a residue equal to one, for the diagonal propagator and vanishing mixing when the physical particle is on-shell leads to

$$\begin{aligned} \text{Re}\hat{\Sigma}'_{ii}(M_{i,\text{tree}}^2) &= 0 \quad \text{with} \quad \frac{\partial \hat{\Sigma}_{ii}(q^2)}{\partial q^2} = \hat{\Sigma}'_{ii}(q^2), \\ \text{Re}\hat{\Sigma}'_{ij}(M_{i,\text{tree}}^2) &= \text{Re}\hat{\Sigma}'_{ij}(M_{j,\text{tree}}^2) = 0 \quad i \neq j. \end{aligned} \quad (4.28)$$

In our renormalisation programme, Eqs. (4.28) set the field renormalisation constants and avoid having to include corrections on the external legs. The field renormalisation constants are therefore not necessarily gauge invariant nor gauge parameter independent.

#### 4.7.2 Renormalisation conditions and corrections on the mass parameters

As we have explained earlier one needs to fix the counterterms for  $\delta M_{A^0}^2$  and  $\delta t_\beta$  once tadpole renormalisation has been carried through to arrive at finite and gauge invariant  $S$ -matrix elements. Taking  $M_{A^0}$  as an input parameter means that its mass is fixed the same at all orders, we therefore set

$$\delta M_{A^0}^2 = -\text{Re}\Sigma_{A^0 A^0}(M_{A^0}^2). \quad (4.29)$$

Finding a condition to fix  $\delta t_\beta$  is an arduous task that has been debated for sometime. We will study many schemes for  $\delta t_\beta$  in Section 5.

The charged Higgs mass is independent of  $t_\beta$ , it gets a finite correction at one-loop once  $M_{A^0}$  is used as an input parameter

$$M_{H^\pm, \text{1loop}}^2 = M_{H^\pm, \text{tree}}^2 + \text{Re}\Sigma_{H^\pm H^\pm}(M_{H^\pm, \text{tree}}^2) - \text{Re}\Sigma_{A^0 A^0}(M_{A^0}^2) - \text{Re}\Pi_{W^\pm}^T(M_{W^\pm}^2), \quad (4.30)$$

we have used  $\delta M_{W^\pm}^2 = \text{Re}\Pi_{W^\pm}^T(M_{W^\pm}^2)$  where  $\Pi_{W^\pm}^T(q^2)$  is the transverse 2-point function of the  $W^\pm$  following the same implementation as performed in [9]. The finiteness of the corrected charged Higgs mass is the first non trivial check on the code as concerns the Higgs sector.

The sum rule involving the CP-even Higgs masses Eq. (2.14) is also independent of  $t_\beta$ . This sum rule gets corrected at one-loop

$$\begin{aligned} M_{h^0, \text{1loop}}^2 + M_{H^0, \text{1loop}}^2 &= M_{A^0}^2 + M_{Z^0}^2 + \text{Re}\Sigma_{h^0 h^0}(M_{h^0}^2) + \text{Re}\Sigma_{H^0 H^0}(M_{H^0}^2) \\ &+ \frac{g}{2M_{W^\pm}} \left( c_{\alpha-\beta} \delta T_{H^0} - s_{\alpha-\beta} \delta T_{h^0} \right) - \text{Re}\Sigma_{A^0 A^0}(M_{A^0}^2) - \text{Re}\Pi_{Z^0 Z^0}^T(M_{Z^0}^2). \end{aligned} \quad (4.31)$$

Here also we have used  $\delta M_{Z^0}^2 = \text{Re}\Pi_{Z^0 Z^0}^T(M_{Z^0}^2)$  where  $\Pi_{Z^0 Z^0}^T(q^2)$  is the transverse 2-point function of the  $Z^0$  boson, see [9]. Otherwise to predict  $M_{h^0, \text{1loop}}^2$  or  $M_{H^0, \text{1loop}}^2$  one needs a prescription on  $\delta t_\beta$ , see Eq. (4.18). Obviously fixing one of these masses, for instance  $M_{H^0}$  in particular in analogy with  $M_{A^0}$ , is a scheme for  $t_\beta$ . In this scheme therefore  $\text{Re}\hat{\Sigma}_{H^0 H^0}(M_{H^0}^2) = 0$  which sets a gauge invariant counterterm for  $t_\beta$ , see Eq. (5.13).

## 4.8 Constraining the field renormalisation constants

We have introduced through the field renormalisation matrices  $Z_P, Z_C, Z_S$  a total of 12 such constants, see Eq. (4.19). However as argued repeatedly, some of these constants are only involved in the transition involving an external Goldstone bosons, *i.e.* in situations that do not correspond to a physical process. Therefore we can give the constants  $\delta Z_{G^0}, \delta Z_{G^\pm}, \delta Z_{A^0 G^0}, \delta Z_{H^\pm G^\pm}$  any value,  $S$ -matrix elements will not depend on these constants. It is therefore easiest to set these 4 constants to 0 in actual calculations and give them arbitrary values in preliminary tests of a calculation of a physical process.

For the transitions involving physical Higgs particles we just go along the general lines described in Section 4.7.1, in order to avoid loop corrections on the external legs. In the following, in order to avoid too much clutter the masses that will appear as argument are the tree-level masses (or the input mass for  $M_{A^0}$ ). The conditions read

$$Re\hat{\Sigma}'_{A^0 A^0}(M_{A^0}^2) = 0, \quad (4.32)$$

$$Re\hat{\Sigma}'_{H^\pm H^\pm}(M_{H^\pm}^2) = 0, \quad (4.33)$$

$$Re\hat{\Sigma}'_{H^0 H^0}(M_{H^0}^2) = 0, \quad (4.34)$$

$$Re\hat{\Sigma}'_{h^0 h^0}(M_{h^0}^2) = 0, \quad (4.35)$$

$$Re\hat{\Sigma}_{H^0 h^0}(M_{H^0}^2) = 0, \quad Re\hat{\Sigma}_{H^0 h^0}(M_{h^0}^2) = 0. \quad (4.36)$$

From these we immediately derive 6 out of the 8 field renormalisation constants in the Higgs sector

$$\delta Z_{A^0} = Re\Sigma'_{A^0 A^0}(M_{A^0}^2), \quad (4.37)$$

$$\delta Z_{H^\pm} = Re\Sigma'_{H^\pm H^\pm}(M_{H^\pm}^2), \quad (4.38)$$

$$\delta Z_{H^0} = Re\Sigma'_{H^0 H^0}(M_{H^0}^2), \quad (4.39)$$

$$\delta Z_{h^0} = Re\Sigma'_{h^0 h^0}(M_{h^0}^2), \quad (4.40)$$

$$\delta Z_{h^0 H^0} = 2 \frac{Re\Sigma_{H^0 h^0}(M_{H^0}^2) + \delta M_{H^0 h^0}^2}{M_{H^0}^2 - M_{h^0}^2}, \quad (4.41)$$

$$\delta Z_{H^0 h^0} = 2 \frac{Re\Sigma_{H^0 h^0}(M_{h^0}^2) + \delta M_{H^0 h^0}^2}{M_{h^0}^2 - M_{H^0}^2}. \quad (4.42)$$

When considering a process with  $A^0$  as an external leg<sup>\*</sup>, in principle it involves the  $A^0 \rightarrow A^0$  transition but also the  $A^0 \rightarrow Z^0$  and the  $A^0 \rightarrow G^0$  transitions. The field renormalisation constant  $\delta Z_{A^0}$ , see Eq. (4.37) allows to set the  $A^0 \rightarrow A^0$  transition to 0 and moves its effect to a vertex counterterm correction. One therefore would be tempted by setting  $\hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) = 0$  together with  $\hat{\Sigma}_{A^0 G^0}(M_{A^0}^2) = 0$  as is done almost everywhere in the literature. In our case this would mean that the remaining constant  $\delta Z_{G^0 A^0}$  could be derived equivalently from one of these conditions. However the Ward identity we derived in Eq. (4.24) imposes a very important constraint. It shows that in a general non-linear gauge we can not impose both  $\hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) = 0$  and  $\hat{\Sigma}_{A^0 G^0}(M_{A^0}^2) = 0$ . It looks at first sight that this requires that one introduces loop corrections on the external legs when considering for example processes with the pseudoscalar Higgs as an external leg. In the linear gauge on the other hand this is possible since  $\mathcal{F}_{GA}^{\epsilon, \tilde{\gamma}}(q^2) = 0$ , we could then adjust  $\delta Z_{G^0 A^0}$  and  $\delta Z_{A^0 G^0}$  to have  $\hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) = 0$  and  $\hat{\Sigma}_{A^0 G^0}(M_{A^0}^2) = 0$ . Note however that contrary to what we encounter in some publications, see for example [16, 17],  $q^2 \hat{\Sigma}_{A^0 Z^0}(q^2) + M_{Z^0} \hat{\Sigma}_{A^0 G^0}(q^2)$  does not vanish for any value of  $q^2$  but only for  $q^2 = M_{A^0}^2$ <sup>†</sup>.

Let us show how despite the constraint in Eq. (4.24) we can still avoid one-loop corrections and counterterms in the external legs associated with an external pseudoscalar  $A^0$ . Of concern to

<sup>\*</sup>The argument with the charged Higgs is exactly the same, therefore we will not make explicit the detailed derivation of the field renormalisation constant  $\delta Z_{G^\pm H^\pm}$  but only quote the result.

<sup>†</sup>The charged counterpart of this identity is also not valid for *any*  $q^2$  as is assumed sometimes, see [18].



us are the transition  $A^0 - Z^0$  and  $A^0 - G^0$ . The idea is that although we can not make both  $\hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) = 0$  and  $\hat{\Sigma}_{A^0 G^0}(M_{A^0}^2) = 0$ , we will try to make the combined contribution to the external leg vanish. This combined contribution is pictured in Fig. 1.

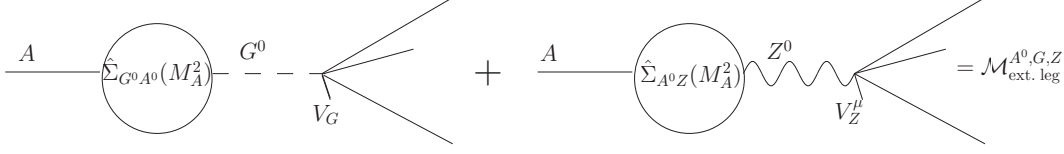


Figure 1: *The combined contribution of the  $A^0 - Z^0$  and  $A^0 - G^0$  transitions*

To the tree-level coupling of the  $A^0$  to some vertex  $V$ , at one-loop the transition  $A^0 - G^0$  involves the coupling of the tree-level neutral Goldstone to this vertex,  $V_G$  while the  $Z^0$  transition involves the corresponding vertex  $V_Z^\mu$ . The total contribution of Fig. 1 for  $A^0$  with momentum  $q$  on-shell with  $q^2 = M_{A^0}^2$  writes

$$\begin{aligned} \mathcal{M}_{\text{ext. leg}}^{A^0, G, Z} &= \frac{\hat{\Sigma}_{A^0 G^0}(M_{A^0}^2)V_G + q \cdot V_Z \hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2)}{M_{A^0}^2 - M_{Z^0}^2} \\ &= \frac{V_G}{M_{A^0}^2 - M_{Z^0}^2} \left( \hat{\Sigma}_{A^0 G^0}(M_{A^0}^2) + M_{Z^0} \hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) \right). \end{aligned} \quad (4.43)$$

In the second step of Eq. (4.43) we used another identity that can be readily derived at tree-level from the invariance of the Lagrangian under gauge transformations<sup>†</sup>. Therefore in order not to deal with any correction on the external pseudo-scalar leg we require

$$\hat{\Sigma}_{A^0 G^0}(M_{A^0}^2) + M_{Z^0} \hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) = 0. \quad (4.44)$$

For this requirement Eq. (4.44), which is a renormalisation condition, to be consistent with the Ward identity in Eq. (4.24) leads to

$$\hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) = -\frac{1}{M_{Z^0}} \hat{\Sigma}_{A^0 G^0}(M_{A^0}^2) = \frac{1}{(4\pi)^2} \frac{e^2 M_{Z^0}}{s_{2W}^2} s_{2\beta} \mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(M_{A^0}^2). \quad (4.45)$$

In particular with  $\mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(M_{A^0}^2) = 0$  in the linear gauge, we can make  $\hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) = \hat{\Sigma}_{A^0 G^0}(M_{A^0}^2) = 0$ . This condition readily gives

$$\delta Z_{G^0 A^0} = -s_{2\beta} \frac{\delta t_\beta}{t_\beta} - 2 \frac{\Sigma_{A^0 Z^0}^{\text{tad}}(M_{A^0}^2)}{M_{Z^0}} + \frac{2}{(4\pi)^2} \frac{e^2}{s_{2W}^2} s_{2\beta} \mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(M_{A^0}^2). \quad (4.46)$$

Since  $\delta Z_{A^0 G^0}$  only enters in off-shell processes,  $A^0$  off-shell or an external Goldstone boson, there is no need to constrain it through some other renormalisation condition. Our aim, as stressed repeatedly, is not to renormalise all Green's functions, but only S-matrix elements without the need for external leg corrections. The Ward identities that we derived in this section were, numerically, checked extensively in our code for various values of  $q^2$  including  $q^2 = M_{A^0}^2$  and  $q^2 = M_{Z^0}^2$  and for different values of the non-linear gauge parameters. Moreover it is thanks to the  $\mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(M_{A^0}^2)$  contribution in  $\delta Z_{G^0 A^0}$  that we are able to obtain finite and gauge invariant results for processes involving  $A^0$  as an external particle. For  $\delta Z_{G^\pm H^\pm}$  a similar derivation gives

$$\delta Z_{G^\pm H^\pm} = -s_{2\beta} \frac{\delta t_\beta}{t_\beta} - 2 \frac{\Sigma_{H^\pm W^\pm}^{\text{tad}}(M_{H^\pm}^2)}{M_{W^\pm}} + \frac{2}{(4\pi)^2} \frac{e^2}{s_{2W}^2} \mathcal{G}_{HW}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(M_{H^\pm}^2). \quad (4.47)$$

<sup>†</sup>Consider the part of the Lagrangian with the  $Z^0$  and the neutral Goldstone  $G^0$ . Before gauge-fixing this Lagrangian is invariant under the transformation  $Z_\mu^0 \rightarrow Z_\mu^0 + i\partial_\mu \omega$ ,  $G^0 \rightarrow G^0 + M_{Z^0} \omega$ . If the  $Z^0$  (vector) current is  $V_Z^\mu$  and the Goldstone current  $V_G$ , that is we have the interaction  $Z^0 \cdot V_Z + G^0 V_G$ , invariance of the Lagrangian implies  $-i\partial_\alpha V_Z^\alpha + M_{Z^0} V_G = 0$ . In Eq. (4.43), this implies  $q \cdot V_Z = M_{Z^0} V_G$  where  $q$  is the  $Z^0$  momentum.

With  $\delta Z_{G^0 A^0}$  (and  $\delta Z_{G^\pm H^\pm}$ ) all our field renormalisation constants are set and defined.

## 5 Definitions of $t_\beta$ and the $t_\beta$ schemes

### 5.1 Dabelstein-Chankowski-Pokorski-Rosiek Scheme (DCPR)

This scheme, which we will refer to as the DCPR scheme, has been quite popular and is based on an OS renormalisation scheme in the Higgs sector [14, 15] working in the usual linear gauge. The definition of  $t_\beta$  however is difficult to reconcile with an On-Shell quantity that represents a direct interpretation in terms of a physical observable. One first introduces a wave function renormalisation constant,  $\delta Z_{H_i}$ , for each Higgs doublet  $H_i$ , *i.e.* before rotation

$$H_i \rightarrow (1 + \frac{1}{2}\delta Z_{H_i})H_i \quad i = 1, 2. \quad (5.1)$$

To make contact with our approach and parameters, as concerns wave function renormalisation, we refer to Appendix B. The vacuum expectation values are also shifted such that the counterterm for each  $v_i$  writes

$$v_i \rightarrow v_i \left( 1 - \frac{\tilde{\delta} v_i}{v_i} + \frac{1}{2}\delta Z_{H_i} \right), \quad (5.2)$$

giving

$$\frac{\delta t_\beta}{t_\beta} = \frac{\tilde{\delta} v_1}{v_1} - \frac{\tilde{\delta} v_2}{v_2} - \frac{1}{2}(\delta Z_{H_1} - \delta Z_{H_2}). \quad (5.3)$$

The DCPR scheme takes  $\frac{\tilde{\delta} v_1}{v_1} = \frac{\tilde{\delta} v_2}{v_2}$  such that in effect

$$\frac{\delta t_\beta}{t_\beta} = \frac{1}{2}(\delta Z_{H_2} - \delta Z_{H_1}). \quad (5.4)$$

$t_\beta$  is defined by requiring that the (renormalised)  $A^0 Z^0$  transition vanish at  $q^2 = M_{A^0}^2$ , therefore from

$$Re \hat{\Sigma}_{A^0 Z^0}(M_{A^0}^2) = 0, \quad (5.5)$$

with

$$\hat{\Sigma}_{A^0 Z^0}(q^2) = \Sigma_{A^0 Z^0}(q^2) + \frac{M_{Z^0}}{4} s_{2\beta} (\delta Z_{H_2} - \delta Z_{H_1} + 2 \frac{\delta t_\beta}{t_\beta}), \quad (5.6)$$

one obtains that

$$\frac{\delta t_\beta}{t_\beta}^{\text{DCPR}} = - \frac{1}{M_{Z^0} s_{2\beta}} Re \Sigma_{A^0 Z^0}(M_{A^0}^2). \quad (5.7)$$

This definition is clearly not directly related to an observable. Moreover  $\delta t_\beta$  is expressed in terms of wave function renormalisation constants, see Eq. (5.4).

### 5.2 $\overline{\text{DR}}$ Scheme ( $\overline{\text{DR}}$ )

In this scheme the counterterm for  $t_\beta$  is taken to be a pure divergence proportional to the ultraviolet (UV) factor in dimensional reduction,  $C_{UV}$

$$C_{UV} = 2/(4-n) - \gamma_E + \ln(4\pi), \quad (5.8)$$

where  $n$  is the dimensionality of space-time. In this scheme the finite part of the counterterm is therefore set zero:

$$\frac{\delta t_\beta^{\text{fin}}}{t_\beta} = 0. \quad (5.9)$$

The divergent part can be related to a few quantities not necessarily directly related to an observable. In the vein of the DCPR approach within the linear gauge where  $\delta t_\beta$  is defined in Eq. (5.4), solving for  $\delta Z_{H_2} - \delta Z_{H_1}$  leads to the HHW prescription of Hollik, Heinemeyer and Weiglein [19], see also Eq. (B.15),

$$\frac{\delta t_\beta}{t_\beta}^{\overline{\text{DR}}-\text{HHW}} = \frac{1}{2c_{2\alpha}} (Re\Sigma'_{h^0 h^0}(M_{h^0}^2) - Re\Sigma'_{H^0 H^0}(M_{H^0}^2))^\infty. \quad (5.10)$$

The superscript  $^\infty$  means that only the infinite  $C_{UV}$  part in dimensional reduction is taken into account. A more satisfactory  $\overline{\text{DR}}$  scheme can be based on a physical observable. Pierce and Papadopoulos [20] have defined  $\delta t_\beta$  by relating it to the *divergent* part of  $M_{H^0}^2 - M_{h^0}^2$ . Note that the sum  $M_{H^0}^2 + M_{h^0}^2$  does not depend on  $t_\beta$  as can be seen from the tree-level sum rule in Eq. (2.14). Hence, see also Eq. (4.31),

$$\begin{aligned} \frac{\delta t_\beta}{t_\beta}^{\overline{\text{DR}}-\text{PP}} &= \frac{1}{2s_{2\beta}s_{2(\alpha+\beta)}M_{Z^0}^2} \left( \frac{1}{v} (c_{\alpha-\beta}(1+2s_{\alpha-\beta}^2)\delta T_{H^0} + s_{\alpha-\beta}(1+2c_{\alpha-\beta}^2)\delta T_{h^0}) \right. \\ &\quad \left. + Re\Sigma_{H^0 H^0}(M_{H^0}^2) - Re\Sigma_{h^0 h^0}(M_{h^0}^2) + c_{2(\alpha+\beta)}Re\Sigma_{A^0 A^0}(M_{A^0}^2) - c_{2(\alpha+\beta)}Re\Pi_{Z^0 Z^0}^T(M_{Z^0}^2) \right)^\infty. \end{aligned} \quad (5.11)$$

### 5.3 An On-Shell Scheme ( $\text{OS}_{M_H}$ ) with $M_{H^0}$ as an input

In this scheme one takes  $M_{H^0}$ , the largest of the two scalar Higgs masses, as an input parameter. This trade-off is operative in the Higgs sector independently of any process. Therefore  $M_{H^0}$  is no longer a prediction but is extracted from a measurement together with  $M_{A^0}$ . As such it does not receive a correction at any loop order,  $\delta t_\beta$  is defined from the constraint

$$Re\hat{\Sigma}_{H^0 H^0}(M_{H^0}^2) = 0, \quad (5.12)$$

which leads to

$$\begin{aligned} \frac{\delta t_\beta}{t_\beta}^{\text{OS}_{M_H}} &= \frac{1}{s_{2\beta}s_{2(\alpha-\beta)}M_{A^0}^2} \left( (c_\alpha^2 - s_\beta^2 s_{\alpha-\beta}^2) \frac{\delta T_{\phi_1^0}}{v_1} + (s_\alpha^2 - c_\beta^2 s_{\alpha-\beta}^2) \frac{\delta T_{\phi_2^0}}{v_2} \right. \\ &\quad \left. + Re\Sigma_{H^0 H^0}(M_{H^0}^2) - s_{\alpha-\beta}^2 Re\Sigma_{A^0 A^0}(M_{A^0}^2) - c_{\alpha+\beta}^2 Re\Pi_{Z^0 Z^0}^T(M_{Z^0}^2) \right). \end{aligned} \quad (5.13)$$

This scheme has been advocated in [17, 13] and is one of the scheme implemented in **SloopS**. At tree-level,  $t_\beta$  is extracted from the relation defined in Eq. (2.16)

$$c_{2\beta}^2 = \frac{(M_{A^0}^2 + M_{Z^0}^2 - M_{H^0}^2)M_{H^0}^2}{M_{A^0}^2 M_{Z^0}^2}. \quad (5.14)$$

In our numerical examples the input parameters are such that the requirement  $c_{2\beta}^2 \leq 1$  is always met. In fact given a set  $M_{A^0}, M_{Z^0}$  we *generate*  $M_{H^0}$  through a given value of  $t_\beta$ . The value  $M_{H^0}$  is taken as the physical mass at all loop orders, in particular at one-loop it does not receive a correction. As pointed out in Section 2, in general with a set  $M_{H^0}, M_{A^0}, M_{Z^0}$   $c_{2\beta}^2 \leq 1$  is not guaranteed. With this important proviso, we extract  $\tan\beta$  (with  $\tan\beta > 1$ ) as

$$t_\beta = \sqrt{\frac{M_{A^0}M_{Z^0} + M_{H^0}\sqrt{M_{A^0}^2 + M_{Z^0}^2 - M_{H^0}^2}}{M_{A^0}M_{Z^0} - M_{H^0}\sqrt{M_{A^0}^2 + M_{Z^0}^2 - M_{H^0}^2}}}. \quad (5.15)$$

That this choice might lead to large corrections and large uncertainty can already be guessed by considering the uncertainty on  $\tan\beta$  given an uncertainty on  $M_{H^0}, M_{A^0}, M_{Z^0}$  with respectively  $\delta M_{H^0}, \delta M_{A^0}, \delta M_{Z^0}$ . For clarity let us take  $\delta M_{Z^0} = 0$  as would be fit from an experimental point of view since  $M_{Z^0}$  is known with an excellent precision from the LEP measurements. We find

$$\frac{\delta t_\beta}{t_\beta} = \frac{M_{A^0}^2}{M_{H^0}^2 - M_{A^0}^2} \frac{M_{H^0}^2}{M_{H^0}^2 - M_{Z^0}^2} \left( -\frac{M_{H^0}^2 - M_{Z^0}^2}{M_{A^0}^2} \frac{\delta M_{A^0}^2}{M_{A^0}^2} + \frac{M_{H^0}^2}{M_{A^0}^2} \frac{2M_{H^0}^2 - M_{A^0}^2 - M_{Z^0}^2}{M_{H^0}^2} \frac{\delta M_{H^0}^2}{M_{H^0}^2} \right) \quad (5.16)$$

With typical input parameters in the decoupling limit  $M_{A^0} \gg M_{Z^0}$  with  $M_{A^0}/M_{H^0} \sim 1$  a large uncertainty ensues, to wit

$$\frac{\delta t_\beta}{t_\beta} \simeq \frac{1}{M_{H^0}^2/M_{A^0}^2 - 1} \left( -\frac{\delta M_{A^0}^2}{M_{A^0}^2} + \frac{\delta M_{H^0}^2}{M_{H^0}^2} \right). \quad (5.17)$$

Therefore although  $\delta t_\beta$  is manifestly gauge invariant one should expect large uncertainty from loop corrections. This scheme is similar to the one considered in [6] based on Eq. (5.14).

## 5.4 $A_{\tau\tau}$ as an input parameter (OS $_{A\tau\tau}$ )

$\beta$  which appears in the Higgs sector relies on the assumption of a basis, only quantities which are basis independent are physical quantities [8, 21]. The Higgs potential of the MSSM appears as a general two-Higgs doublet model if one restricts oneself solely to the Higgs sector. The degeneracy is lifted when defining the Yukawa Higgs coupling to fermions. This picks up a specific direction. One should therefore define  $\tan\beta$  from the Higgs couplings to fermions. Since  $M_{A^0}$  is used as an input parameter assuming one has had access to the pseudoscalar Higgs, it looks natural to take a coupling  $A^0 f \bar{f}$ . Since couplings to quarks are subject to large QCD radiative corrections the best choice is to consider the  $A_{\tau\tau}$  coupling which is the largest coupling to leptons,

$$\mathcal{L}_{A\tau\tau}^0 = i \frac{m_\tau}{v_1} s_\beta \bar{\tau} \gamma_5 \tau A^0 = i \frac{gm_\tau}{2M_{W^\pm}} t_\beta \bar{\tau} \gamma_5 \tau A^0 \quad \text{with} \quad v_1 = v c_\beta. \quad (5.18)$$

This coupling can be extracted from the measurement of the width  $\Gamma_{A\tau\tau}$  with  $m_\tau$  the mass of the  $\tau$ . Note also that  $\delta\Gamma_{A\tau\tau} = 2\delta t_\beta/t_\beta$  so that contrary to the On-Shell scheme based on  $M_{H^0}$ , OS $_{M_H}$ , this scheme should therefore not introduce additional large uncertainties assuming of course that this decay can be large and be measured precisely. This scheme appears therefore very natural, however it has not been used in practice because one has considered it as being a *process dependent* definition set outside the purely Higgs sector which moreover implies that fixing the counterterm involves a three-point function. This last argument is unjustified, take for example the  $G_\mu$  scheme in the SM where muon decay is used as a trade-off for  $M_{W^\pm}$  taking advantage of the fact that  $G_\mu$  has been for a long time so much better measured than  $M_{W^\pm}$ . The  $G_\mu$  scheme involves four-point functions. We find that technically this scheme is not more difficult to implement than a scheme based on two-point functions. The full counterterm to  $A_{\tau\tau}$  involves the  $G^0 \rightarrow A^0$  shift, the  $A^0$  and  $\tau^\pm$  wave function renormalisation constants among other things, we get

$$\begin{aligned} \delta\mathcal{L}_{A\tau\tau} &= \mathcal{L}_{A\tau\tau}^0 \left( \delta_{\text{CT}}^{A\tau\tau} + \frac{\delta t_\beta}{t_\beta} \right) \quad \text{with} \\ \delta_{\text{CT}}^{A\tau\tau} &= \left( \frac{\delta m_\tau}{m_\tau} + \frac{\delta e}{e} + \frac{c_W^2}{2s_W^2} \frac{\delta M_{W^\pm}^2}{M_{W^\pm}^2} - \frac{1}{2s_W^2} \frac{\delta M_{Z^0}^2}{M_{Z^0}^2} + \frac{1}{2} \delta Z_{A^0 A^0} - \frac{1}{2t_\beta} \delta \tilde{Z}_{G^0 A^0} \right. \\ &\quad \left. + \frac{1}{2} (\delta Z_L^\tau + \delta Z_R^\tau) \right), \\ -\frac{1}{2t_\beta} \delta \tilde{Z}_{G^0 A^0} &= \frac{1}{t_\beta} \frac{\Sigma_{A^0 Z^0}(M_{A^0}^2)}{M_{Z^0}} - \frac{1}{1+t_\beta^2} \frac{\alpha}{2\pi} M_{Z^0} \mathcal{F}_{GA}^{\tilde{e}, \tilde{\gamma}}(M_{Z^0}^2). \end{aligned} \quad (5.19)$$

$\delta m_\tau$ , the  $\tau$  mass counterterm,  $\delta e$  the electromagnetic coupling counterterm,  $\delta M_{W^\pm, Z^0}$  the gauge bosons mass counterterms and the  $\tau$  wave function renormalisation constant  $\delta Z_{L,R}^\tau$  counterterms

are defined on-shell exactly as in the SM [9]. The full one-loop virtual corrections consist of the vertex corrections,  $\delta_V^{A\tau\tau}$  which contributes a one-loop vertex correction to the decay rate as:

$$\delta\Gamma_1^{\text{Vertex}} = 2\Gamma_0 \delta_V^{A\tau\tau}. \quad (5.20)$$

The latter are made UV-finite by the addition of the counterterm in Eq. (5.19). These virtual QED corrections, both vertex and counterterm (from  $\delta m_\tau$  and  $\delta Z_{L,R}^\tau$ ) include genuine QED corrections through photon exchange which are infrared divergent. In our case the infrared divergence can be trivially regularised through the introduction of a small fictitious mass,  $\lambda$ , for the photon. As known, the fictitious mass dependence is cancelled when photon bremsstrahlung is added. Taking into account the latter may depend on the experimental set-up that often requires cuts on the additional photon kinematical variables. Therefore it is much more appropriate to take as an observable a quantity devoid of such cuts, knowing that hard/soft radiation can be easily added. Fortunately for a *neutral* decay such as this one which is of an Abelian nature, the virtual QED correction constitutes a gauge-invariant subset that can be trivially calculated separately. The virtual QED corrections to the decay width  $A^0 \rightarrow \tau^+\tau^-$  are known [22], they contribute a one-loop correction

$$\begin{aligned} \delta\Gamma_1^{QED} &= 2\Gamma_0 \delta_v^{QED} \quad \text{with} \\ \delta_v^{QED} &= \frac{\alpha}{2\pi} \left( - \left( \frac{1+\beta^2}{2\beta} \ln \frac{1+\beta}{1-\beta} - 1 \right) \ln \frac{m_\tau^2}{\lambda^2} - 1 \right. \end{aligned} \quad (5.21)$$

$$\left. + \frac{1+\beta^2}{\beta} \left[ \text{Li}_2 \left( \frac{1-\beta}{1+\beta} \right) + \ln \frac{1+\beta}{2\beta} \ln \frac{1+\beta}{1-\beta} - \frac{1}{4} \ln^2 \frac{1+\beta}{1-\beta} + \frac{\pi^2}{3} \right] \right),$$

$$\beta = \sqrt{1 - \frac{4m_\tau^2}{M_{A^0}^2}}, \quad (5.22)$$

$$\text{Li}_2(x) = - \int_0^x \frac{dt}{t} \ln(1-t). \quad (5.23)$$

This QED correction only depends on  $M_{A^0}, e, m_\tau$  as it should and does not involve any other (MSSM) parameter. Subtracting this QED correction from the full one-loop virtual correction in Eq. (5.19) will give the genuine SUSY non QED contribution that does not depend on any fictitious photon mass nor any experimental cut. Our scheme is to require that  $\delta t_\beta$  is such that this contribution vanishes and that therefore  $A^0 \rightarrow \tau^+\tau^-$  is only subject to QED corrections. This gives

$$\frac{\delta t_\beta}{t_\beta}^{\text{OS}_{A\tau\tau}} = - \left( \delta_V^{A\tau\tau} + \delta_{\text{CT}}^{A\tau\tau} - \delta_v^{QED} \right). \quad (5.24)$$

This definition is independent of the fictitious mass of the photon  $\lambda$  used as a regulator. We have checked this explicitly within **SloopS**.

## 6 Set-up of the automatic calculation of the cross sections

All the steps necessary for the renormalisation of the Higgs sector as presented here together with a complete definition of the MSSM have been implemented in **SloopS**. As we will discuss in a forthcoming publication [23] the other sectors have also been implemented and results relying on the complete renormalisation of the MSSM have been given in [12]. Since even the calculation of a single two-point function in the MSSM requires the calculation of a hundred of diagrams, some automatisation is unavoidable. Even in the SM, one-loop calculations of  $2 \rightarrow 2$  processes involve hundreds of diagrams and a hand calculation is practically impracticable. Efficient automatic codes for any generic  $2 \rightarrow 2$  process, that have now been exploited for many  $2 \rightarrow 3$  [24, 25] and

even some  $2 \rightarrow 4$  [26, 27] processes, are almost unavoidable for such calculations. For the electroweak theory these are the `GRACE-loop` [9] code and the bundle of packages based on `FeynArts` [28], `FormCalc` [29] and `LoopTools` [30], that we will refer to as FFL for short.

With its much larger particle content, far greater number of parameters and more complex structure, the need for an automatic code at one-loop for the Minimal Supersymmetric Standard Model is even more of a must. A few parts that are needed for such a code have been developed based on an extension of [31] but, as far as we know, no complete code exists or is, at least publicly, available. `Grace-susy` [32] is now also being developed at one-loop and many results exist [13]. One of the main difficulties that has to be tackled is the implementation of the model file, since this requires that one enters the thousands of vertices that define the Feynman rules. On the theory side a proper renormalisation scheme needs to be set up, which then means extending many of these rules to include counterterms. When this is done one can just use, or hope to use, the machinery developed for the SM, in particular the symbolic manipulation part and most importantly the loop integral routines including tensor reduction algorithms or any other efficient set of basis integrals.

`SloopS` combines `LANHEP` [33] (originally part of the package `COMPHEP` [34]) with the FFL bundle but with an extended and adapted `LoopTools` [11]. `LANHEP` is a very powerful routine that *automatically* generates all the sets of Feynman rules of a given model, the latter being defined in a simple and compact format very similar to the canonical coordinate representation. Use of multiplets and the superpotential is built-in to minimize human error. The ghost Lagrangian is derived directly from the BRST transformations. The `LANHEP` module also allows to shift fields and parameters and thus generates counterterms most efficiently. Understandably the `LANHEP` output file must be in the format of the model file of the code it is interfaced with. In the case of `FeynArts` both the *generic* (Lorentz structure) and *classes* (particle content) files had to be given. Moreover, because we use a non-linear gauge fixing condition [9], see below, the `FeynArts` default *generic* file had to be extended.

## 7 $t_\beta$ scheme dependence of physical observables, gauge invariance: A numerical investigation

In this first investigation we will restrict ourselves to Higgs observables. Other observables involving other supersymmetric particles require that we first expose and detail our renormalisation procedure of the chargino/neutralino and the sfermion sector. This will be presented in [23]. We have however presented some results on the  $\tan\beta$  scheme dependence of a few cross sections that are needed for the calculation of the relic density in the MSSM [12].

### 7.1 Parameters

To make contact with the analysis of [6] and also allow comparisons we will consider the 3 sets of benchmark points for the Higgs based on [35]. The 3 sets of parameters called *mhmax*, *large  $\mu$*  and *nomix* are as in [35] except that we set a common tri-linear  $A_f$  to all sfermions for convenience. For each set there are two values of  $t_\beta$ ,  $t_\beta = 3, 50$ .

### 7.2 Gauge independence and the finite part of $t_\beta$

If  $t_\beta$  is defined as a physical parameter then  $\delta t_\beta$  must be gauge invariant and gauge parameter independent. Our non-linear gauge fixing allows us to check the gauge parameter independence of  $\delta t_\beta$  and hence  $t_\beta$ . Even when two schemes are gauge parameter independent the values of  $\delta t_\beta$

Parameter	Value	Parameter	Value	Constant	Value
$s_W$	0.48076	$m_\mu$	0.1057	$m_s$	0.2
$e$	0.31345	$m_\tau$	1.777	$m_t$	174.3
$g_s$	1.238	$m_u$	0.046	$m_b$	3
$M_{Z^0}$	91.1884	$m_d$	0.046	$M_{A^0}$	500
$m_e$	0.000511	$m_c$	1.42	$t_\beta$	3;50

$mhmax$	Value	$nomix$	Value	$large \mu$	Value
$\mu$	-200	$\mu$	-200	$\mu$	1000
$M_2$	200	$M_2$	200	$M_2$	400
$M_3$	800	$M_3$	800	$M_3$	200
$M_{\tilde{F}_L}$	1000	$M_{\tilde{F}_L}$	1000	$M_{\tilde{F}_L}$	400
$M_{\tilde{f}_R}$	1000	$M_{\tilde{f}_R}$	1000	$M_{\tilde{f}_R}$	400
$A_f$	$2000 + \mu/t_\beta$	$A_f$	$\mu/t_\beta$	$A_f$	$-300 + \mu/t_\beta$

Table 1: The set of SM and MSSM parameters for the benchmark points. All mass parameters are in GeV. We take  $M_1$  according to the so-called gaugino mass unification with  $M_1 = \frac{5s_W^2 M_2}{3c_W^2}$ .

are not expected to be the same. It is therefore also interesting to inquire how much two schemes differ from each other. Naturally since  $\delta t_\beta$  is not ultraviolet finite we split this contribution into a finite part and infinite part, the latter being regularised in dimensional reduction, such that

$$\delta t_\beta = \delta t_\beta^{\text{fin}} + \delta t_\beta^\infty C_{UV} . \quad (7.1)$$

The  $\overline{\text{DR}}$  schemes have by definition  $\delta t_\beta^{\text{fin}} = 0$ . When calculating observables in this scheme we will also need to specify a scale  $\bar{\mu}$  which we associate with the scale introduced by dimensional reduction. For the latter our default value is  $\bar{\mu} = M_{A^0}$ . Our set of non-linear gauge parameters is defined as  $\text{nlgs} = (\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\omega}, \tilde{\rho}, \tilde{\kappa}, \tilde{\epsilon}, \tilde{\gamma})$ .

The usual linear gauge,  $\text{nlgs} = 0$ , corresponds to all these parameters set to 0. For the gauge parameter independence we will compare the results of the linear gauge to a non-linear gauge where all the non-linear gauge parameters have been set to 10, referring to this as  $\text{nlgs} = 10$ .

To make the point about the gauge parameter dependence it is enough to consider only one of the benchmarks points.

$\delta t_\beta^\infty$	$\text{nlgs} = 0$	$\text{nlgs} = 10$	$\delta t_\beta^{\text{fin}}$	$\text{nlgs} = 0$	$\text{nlgs} = 10$
DCPR	$-3.19 \times 10^{-2}$	$-1.04 \times 10^{-1}$	DCPR	-0.10	-0.27
$\text{OS}_{M_H}$	$-3.19 \times 10^{-2}$	$-3.19 \times 10^{-2}$	$\text{OS}_{M_H}$	+0.92	+0.92
$\text{OS}_{A\tau\tau}$	$-3.19 \times 10^{-2}$	$-3.19 \times 10^{-2}$	$\text{OS}_{A\tau\tau}$	-0.10	-0.10
$\overline{\text{DR}}\text{-HHW}$	$-3.19 \times 10^{-2}$	$+5.32 \times 10^{-2}$	$\overline{\text{DR}}\text{-HHW}$	0	0
$\overline{\text{DR}}\text{-PP}$	$-3.19 \times 10^{-2}$	$-3.19 \times 10^{-2}$	$\overline{\text{DR}}\text{-PP}$	0	0

Table 2: Gauge dependence of  $\delta t_\beta$  at the scale  $\bar{\mu} = M_{A^0}$  for the set  $mhmax$  at  $t_\beta = 3$ .

As expected we see from Table 2 that only the schemes based on a physical definition of  $t_\beta$  are gauge parameter independent. Therefore neither DCPR nor a  $\overline{\text{DR}}$  manifestation of it based on [19] are gauge independent. Within the physical definitions note that although the divergent part is, as expected, the same for all the schemes in all gauges, the finite parts are quite different from each other, in particular the  $\text{OS}_{M_H}$  scheme introduces a “correction” of about 30% to  $t_\beta$ . This is just an indication that this scheme might induce large corrections on observables. However one needs to be cautious, in the same way that the  $C_{UV}$  part cancels in observables, a large finite correction could, in principle, also be absorbed when we consider a physical process. Our rather



extensive analysis will show that this is, after all, not the case. Schemes where the finite part of  $\delta t_\beta$  is large do, generally, induce large corrections. It is important to note that for the linear gauge all schemes give the same  $C_{UV}$  part. Having made the point about the gauge parameter dependence, we will now work purely in the linear gauge since some of the schemes introduced in the literature are *acceptable* only within the linear gauge. Therefore in this case the results for  $\overline{\text{DR}}$ -HHW and  $\overline{\text{DR}}$ -PP are the same and will be denoted as  $\overline{\text{DR}}$  in what follows.

### 7.3 $\delta t_\beta^{fin}$

$t_\beta = 3$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>	$t_\beta = 50$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
DCPR	-0.10	-0.06	-0.08	DCPR	+3.42	+14.57	+0.48
OS $_{M_H}$	+0.92	-1.31	+0.64	OS $_{M_H}$	-385.53	-2010.84	-290.18
OS $_{A_{\tau\tau}}$	-0.10	-0.06	-0.08	OS $_{A_{\tau\tau}}$	+0.12	-4.72	+0.16
$\overline{\text{DR}}$	0	0	0	$\overline{\text{DR}}$	0	0	0

Table 3:  $\delta t_\beta^{fin}$  for the Higgs benchmark points.

First of all let us mention that our numerical results concerning the DCPR and  $\overline{\text{DR}}$  schemes agree quite well with those of [6] concerning the shifts in  $t_\beta$  and the lightest CP-even Higgs mass. Our results for OS $_{M_H}$  follow sensibly the same trend as the scheme defined as the Higgs mass scheme in [6]. We see that for small  $t_\beta$  DCPR and OS $_{A_{\tau\tau}}$  give sensibly the same result with a finite relative shift of a few percent. For larger  $t_\beta$  the difference is much larger, we notice that OS $_{A_{\tau\tau}}$  gives much smaller shifts. On the other hand the OS $_{M_H}$  gives huge corrections for  $t_\beta = 50$  well above 100%. As we will see this will have an impact on the radiative corrections on some observables based on this scheme.

### 7.4 Higgs masses and their scheme dependence

$t_\beta = 3$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>	$t_\beta = 50$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$M_{h^0}^{TL} = 72.51$				$M_{h^0}^{TL} = 91.11$			
DCPR	134.28	97.57	112.26	DCPR	144.50	35.88	124.80
OS $_{M_H}$	140.25	86.68	117.37	OS $_{M_H}$	143.76	13.21	124.16
OS $_{A_{\tau\tau}}$	134.25	97.59	112.27	OS $_{A_{\tau\tau}}$	144.50	35.73	124.80
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	134.87	98.10	112.86	$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	144.50	35.77	124.80
$\overline{\text{DR}} \ \bar{\mu} = M_t$	134.47	97.55	112.38	$\overline{\text{DR}} \ \bar{\mu} = M_t$	144.50	35.77	124.80

Table 4: Mass of the lightest CP-even Higgs at one loop in different schemes All masses are in GeV.

We start with the one-loop correction to the lightest CP-even Higgs. Of course, this has now been calculated beyond one-loop as the one-loop correction is large, however a study of the scheme dependence is important. Moreover this study represents a direct application of the code that can be compared to results in the literature. We note that all schemes apart from OS $_{M_H}$  are in very good agreement with each other for both values of  $t_\beta$ . Leaving aside the case of  $t_\beta = 50$  in the large  $\mu$  scenario, despite the very large shifts we observed in  $\delta t_\beta^{fin}$  for the OS $_{M_H}$  scheme, the  $t_\beta$  dependence is much suppressed such that the OS $_{M_H}$  scheme compares favourably with the other schemes. In the case of the correction to the heaviest CP-even Higgs at one loop, by definition there is no correction in the OS $_{M_H}$  scheme, the other schemes agree with each other at a very high level of precision. Moreover especially at high  $t_\beta$  the correction is very small.



$t_\beta = 3$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>	$t_\beta = 50$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$M_{H^0}^{TL} = 503.05$				$M_{H^0}^{TL} = 500.01$			
DCPR	504.68	501.05	504.21	DCPR	499.80	498.90	499.85
OS <sub>M<sub>H</sub></sub>	503.05	503.05	503.05	OS <sub>M<sub>H</sub></sub>	500.01	500.01	500.01
OS <sub>A<sub><math>\tau\tau</math></sub></sub>	504.68	501.05	504.21	OS <sub>A<sub><math>\tau\tau</math></sub></sub>	499.80	498.91	499.85
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	504.52	500.95	504.08	$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	499.80	498.91	500.01
$\overline{\text{DR}} \ \bar{\mu} = M_t$	504.63	501.05	504.19	$\overline{\text{DR}} \ \bar{\mu} = M_t$	499.80	498.91	499.85

Table 5: Mass of the heaviest CP-even Higgs at one loop in different schemes. All masses are in GeV. The one-loop result is based on the relation  $M_{h^0}^2 = M_{h^0, \text{tree}}^2 + \text{Re}\hat{\Sigma}(M_{h^0, \text{tree}}^2)$ .

The mass of the charged Higgs does not depend on  $\delta t_\beta$ , therefore the correction is scheme independent, with the counterterm  $\delta M_{H^\pm}^2 = \delta M_{W^\pm}^2 + \delta M_{A^0}^2$ .

## 7.5 Higgs decays to SM particles and their scheme dependence

### 7.5.1 $A^0 \rightarrow \tau^+ \tau^-$ , the non QED one-loop corrections

$t_\beta = 3$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$\Gamma^{TL} = 9.40 \times 10^{-3}$			
DCPR	$+3.56 \times 10^{-5}$	$-8.71 \times 10^{-6}$	$-7.37 \times 10^{-6}$
OS <sub>M<sub>H</sub></sub>	$+6.41 \times 10^{-3}$	$-7.82 \times 10^{-3}$	$+4.56 \times 10^{-3}$
OS <sub>A<sub><math>\tau\tau</math></sub></sub>	0	0	0
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	$+6.51 \times 10^{-4}$	$+3.94 \times 10^{-4}$	$+5.18 \times 10^{-4}$
$\overline{\text{DR}} \ \bar{\mu} = M_t$	$+2.30 \times 10^{-4}$	$-2.66 \times 10^{-5}$	$+9.67 \times 10^{-5}$
$t_\beta = 50$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$\Gamma^{TL} = 2.61 \times 10^0$			
DCPR	$+3.45 \times 10^{-1}$	$+2.01 \times 10^0$	$+3.35 \times 10^{-2}$
OS <sub>M<sub>H</sub></sub>	$-4.03 \times 10^1$	$-2.09 \times 10^2$	$-3.03 \times 10^1$
OS <sub>A<sub><math>\tau\tau</math></sub></sub>	0	0	0
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	$-1.21 \times 10^{-2}$	$+4.92 \times 10^{-1}$	$-1.66 \times 10^{-2}$
$\overline{\text{DR}} \ \bar{\mu} = M_t$	$-3.00 \times 10^{-2}$	$+4.75 \times 10^{-1}$	$-3.44 \times 10^{-2}$

Table 6: Corrections to the decay  $A^0 \rightarrow \tau^+ \tau^-$  at one loop without the universal QED correction. All widths in GeV.

We now study the non QED corrections to the decay width  $A^0 \rightarrow \tau^+ \tau^-$ , see Section 5.4 for our benchmark points. By definition there is no correction in the OS<sub>A <sub>$\tau\tau$</sub></sub>  scheme. Many interesting and important conclusions can be drawn from Table 6. First of all we note that the scheme dependence is quite large here. After all this is an observable which is directly proportional to  $\delta t_\beta$ . In fact the difference between schemes can be accounted for by  $2\delta t_\beta$  read off from Table 3. For this decay, the OS<sub>M<sub>H</sub></sub> scheme is totally unsuitable, for  $t_\beta = 3$  the correction are of order 100%, whereas for  $t_\beta = 50$  the one-loop correction is an order of magnitude, at least, larger than the tree-level. Especially for  $t_\beta = 3$  in  $\overline{\text{DR}}$  the scale dependence is not negligible. For example with  $\bar{\mu} = m_t$  in  $\overline{\text{DR}}$  the correction is of order  $\sim 1\%$  and  $5\%$  for  $\bar{\mu} = M_{A^0}$ . The corrections are much smaller in DCPR being at the per-mil level. The scale dependence is much smaller for  $t_\beta = 30$  and the corrections in  $\overline{\text{DR}}$  are now smaller than in DCPR. Note also that in the large  $\mu$  scenario the corrections are large.

### 7.5.2 $H^0 \rightarrow \tau^+\tau^-$ , the non QED one-loop corrections

$t_\beta = 3$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$\Gamma^{TL} = 9.35 \times 10^{-3}$			
DCPR	$-1.09 \times 10^{-4}$	$-7.96 \times 10^{-5}$	$-1.09 \times 10^{-4}$
OS <sub><math>M_H</math></sub>	$+6.28 \times 10^{-3}$	$-7.91 \times 10^{-3}$	$+4.47 \times 10^{-3}$
OS <sub><math>A_{\tau\tau}</math></sub>	$-1.45 \times 10^{-4}$	$-7.09 \times 10^{-5}$	$-1.01 \times 10^{-4}$
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	$+5.08 \times 10^{-4}$	$+3.24 \times 10^{-4}$	$+4.17 \times 10^{-4}$
$\overline{\text{DR}} \ \bar{\mu} = M_t$	$+8.57 \times 10^{-5}$	$-9.75 \times 10^{-5}$	$-4.52 \times 10^{-6}$
$t_\beta = 50$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$\Gamma^{TL} = 2.61 \times 10^0$			
DCPR	$+3.54 \times 10^{-1}$	$+2.02 \times 10^0$	$+4.31 \times 10^{-2}$
OS <sub><math>M_H</math></sub>	$-4.03 \times 10^1$	$-2.09 \times 10^2$	$-3.03 \times 10^1$
OS <sub><math>A_{\tau\tau}</math></sub>	$+9.52 \times 10^{-3}$	$+1.94 \times 10^{-3}$	$+9.55 \times 10^{-3}$
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	$-2.59 \times 10^{-3}$	$+4.94 \times 10^{-1}$	$-7.00 \times 10^{-3}$
$\overline{\text{DR}} \ \bar{\mu} = M_t$	$-2.04 \times 10^{-2}$	$+4.76 \times 10^{-1}$	$-2.49 \times 10^{-2}$

Table 7: Corrections to the decay  $H^0 \rightarrow \tau^+\tau^-$  at one loop without the universal QED correction. All widths in GeV.

Similar conclusions can be drawn from the study of the non-QED corrections to  $H^0 \rightarrow \tau^+\tau^-$ , see Table 7. The QED corrections for this decay can be implemented as in [22]. The only difference is that now there is a correction also in the case of the OS <sub>$A_{\tau\tau}$</sub>  scheme. But as expected this correction is very small for both values of  $t_\beta$ . Note that for  $t_\beta = 50$  the DCPR scheme gives very large corrections in the large  $\mu$  scenario. For this process we have not taken into account the one-loop correction to  $M_{H^0}$ , since as we have seen this correction is very small for all schemes and also because one is much too far from the  $\tau\tau$  threshold,  $M_{H^0} \sim 500 \text{ GeV} \gg 2m_\tau$ , where this effect can play a role.

### 7.5.3 $H^0 \rightarrow Z^0 Z^0$ and $A^0 \rightarrow Z^0 h^0$

$t_\beta = 3$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$\Gamma^{TL} = 8.97 \times 10^{-3}$			
DCPR	$+1.59 \times 10^{-2}$	$-6.32 \times 10^{-3}$	$+8.47 \times 10^{-3}$
OS <sub><math>M_H</math></sub>	$+1.40 \times 10^{-2}$	$-4.00 \times 10^{-3}$	$+7.12 \times 10^{-3}$
OS <sub><math>A_{\tau\tau}</math></sub>	$+1.59 \times 10^{-2}$	$-6.32 \times 10^{-3}$	$+8.47 \times 10^{-3}$
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	$+1.57 \times 10^{-2}$	$-6.44 \times 10^{-3}$	$+8.32 \times 10^{-3}$
$\overline{\text{DR}} \ \bar{\mu} = M_t$	$+1.58 \times 10^{-2}$	$-6.32 \times 10^{-3}$	$+8.44 \times 10^{-3}$
$t_\beta = 50$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$\Gamma^{TL} = 6.40 \times 10^{-5}$			
DCPR	$+2.18 \times 10^{-5}$	$-5.14 \times 10^{-4}$	$+3.89 \times 10^{-5}$
OS <sub><math>M_H</math></sub>	$+1.01 \times 10^{-2}$	$+4.66 \times 10^{-3}$	$+7.81 \times 10^{-4}$
OS <sub><math>A_{\tau\tau}</math></sub>	$+3.02 \times 10^{-5}$	$-4.65 \times 10^{-4}$	$+3.97 \times 10^{-5}$
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	$+3.05 \times 10^{-5}$	$-4.77 \times 10^{-4}$	$+4.01 \times 10^{-5}$
$\overline{\text{DR}} \ \bar{\mu} = M_t$	$+3.09 \times 10^{-5}$	$-4.76 \times 10^{-4}$	$+4.05 \times 10^{-5}$

Table 8: Corrections to the decay  $H^0 \rightarrow \tau^+\tau^-$  at one loop. All widths in GeV.

$H^0 \rightarrow Z^0 Z^0$  was studied by [20] where a large correction was found. We confirm here, see Table 8, that a large correction is indeed induced with the one-loop result of the same order if not exceeding

both at  $t_\beta = 3$  and  $t_\beta = 50$  the tree-level result. This larger correction is not due to the scheme dependence since in this process the latter is very small whereas one sees a large correction with all the schemes. The correction is large because the benchmark points with  $M_{A^0} = 500$  GeV are in the decoupling regime where  $H^0 \rightarrow Z^0 Z^0$  practically vanishes at tree-level. The  $H^0 Z^0 Z^0$  is proportional to  $c_{\beta-\alpha} \sim M_{Z^0}/M_{A^0}$ , the coupling is therefore almost induced at one loop without the  $1/M_{A^0}$  suppression. Here again because  $M_{H^0} \gg 2M_{Z^0}$  the one-loop correction on  $M_{H^0}$  is negligible. Very similar results and conclusions can be drawn for the process  $A^0 \rightarrow Z^0 h^0$ , see Table 9.

$t_\beta = 3$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$\Gamma^{TL} = 9.03 \times 10^{-3}$			
DCPR	$+2.42 \times 10^{-2}$	$+3.86 \times 10^{-3}$	$+1.68 \times 10^{-2}$
OS <sub><math>M_H</math></sub>	$+2.23 \times 10^{-2}$	$+6.20 \times 10^{-3}$	$+1.55 \times 10^{-2}$
OS <sub><math>A_{\tau\tau}</math></sub>	$+2.50 \times 10^{-2}$	$+3.86 \times 10^{-3}$	$+1.64 \times 10^{-2}$
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	$+2.48 \times 10^{-2}$	$+3.74 \times 10^{-3}$	$+1.67 \times 10^{-2}$
$\overline{\text{DR}} \ \bar{\mu} = M_t$	$+2.41 \times 10^{-2}$	$3.87 \times 10^{-3}$	$+1.68 \times 10^{-2}$
$t_\beta = 50$	<i>mhmax</i>	<i>large <math>\mu</math></i>	<i>nomix</i>
$\Gamma^{TL} = 6.30 \times 10^{-5}$			
DCPR	$+2.39 \times 10^{-5}$	$+8.75 \times 10^{-4}$	$+4.31 \times 10^{-5}$
OS <sub><math>M_H</math></sub>	$+1.00 \times 10^{-3}$	$+5.97 \times 10^{-3}$	$+7.74 \times 10^{-4}$
OS <sub><math>A_{\tau\tau}</math></sub>	$+3.48 \times 10^{-5}$	$+9.26 \times 10^{-4}$	$+4.39 \times 10^{-5}$
$\overline{\text{DR}} \ \bar{\mu} = M_{A^0}$	$+3.51 \times 10^{-5}$	$+9.12 \times 10^{-4}$	$+4.43 \times 10^{-5}$
$\overline{\text{DR}} \ \bar{\mu} = M_t$	$+3.30 \times 10^{-5}$	$+9.12 \times 10^{-4}$	$+4.47 \times 10^{-5}$

Table 9: *Corrections to the decay  $A^0 \rightarrow Z^0 h^0$  at one loop. All widths in GeV.*

## 8 Conclusions

The use of the non-linear gauge has allowed us, for the first time, to quantitatively and qualitatively, study different proposals for the ubiquitous parameter  $\tan\beta$  and its effect on the Higgs observables, both the physical Higgs masses as well as their decays. Our first preliminary conclusion is that the scheme based on the extraction and definition of  $\tan\beta$  from a decay such as  $A^0 \rightarrow \tau^+ \tau^-$  is by far the most satisfactory. Not only is this definition directly related to a physical observable and therefore is gauge independent, the functional dependence of the physical width in  $\tan\beta$  is linear and is the same independently of the value of the pseudoscalar Higgs mass. Moreover the definition is clean once we subtract the universal gauge invariant QED correction. The scheme is also most pleasing and satisfactory since it is the one where the observables we have studied show the least corrections, leading therefore to a stable prediction. On this last count the  $\overline{\text{DR}}$  scheme performs almost just as well. However the widely used  $\overline{\text{DR}}$  scheme extracted from the  $A^0 Z^0$  transition is not gauge invariant and therefore terribly unsatisfactory from a theoretical point of view. In the non-linear gauge with a general gauge-fixing set of parameters, the parameter gauge dependence shows up already at one-loop, whereas it has been known that the scheme fails even in the linear gauge but at two-loop [7]. A gauge independent  $\overline{\text{DR}}$  scheme such as the one proposed in [20] is most satisfactory. A scheme based on the usage of  $M_{H^0}$  as an independent parameter from the Higgs sector leads to too large corrections in most of the observables we considered so far. We therefore propose that the decay  $A^0 \rightarrow \tau^+ \tau^-$  be used as a definition of  $\tan\beta$ . This choice assumes that this decay will one day be measured with high enough precision but this depends much on the spectrum of the MSSM. Were it not for the unambiguous extraction of the full QED corrections, the decay of the charged Higgs to  $\tau\nu$  may also qualify as a suitable input parameter, see [36] for prospects on the measurement of this decay. Apart from the discussion on gauge invariance and the issue of the scheme dependence for  $\tan\beta$ , we have shown how a complete

one-loop renormalisation of the MSSM can be automatised and have given results and details as concerns the Higgs sector which is the first step in a successful implementation of this programme.

## **Acknowledgments**

We would first like to thank David Temes whose help was invaluable in the first stages of the project. We also owe much to our friends of the Minami-Tateya group and the developers of the **Grace-SUSY** code, in particular we learned much from Masaaki Kuroda. It is also a pleasure to acknowledge the fruitful discussions with Ben Allanach. This work is supported in part by GDRI-ACPP of the CNRS (France). The work of A.S. is supported by grants of the Russian Federal Agency of Science NS-1685.2003.2 and RFBR 04-02-17448. This work is also part of the French ANR project, **ToolsDMColl**.

## Appendices

### A The Ward-Slavnov-Taylor identity for the transitions $A^0 Z^0$ and $A^0 G^0$

There is an identity relating the  $A^0 Z^0$  and  $A^0 G^0$  transition. This is most useful for  $q^2 = M_{A^0}^2$ . Contrary to what one might see in some papers, the relation is much more complicated for  $q^2 \neq M_{A^0}^2$  and gets more subtle in the case of the non-linear gauge.

The identity can be most easily derived by considering the BRST transformation on the (“ghost”) operator  $\langle 0 | \bar{c}^Z(x) A^0(y) | 0 \rangle = 0$ . We find

$$\delta_{\text{BRS}} \langle 0 | \bar{c}^Z(x) A^0(y) | 0 \rangle = \langle 0 | (\delta_{\text{BRS}} \bar{c}^Z(x)) A^0(y) | 0 \rangle - \langle 0 | \bar{c}^Z(x) (\delta_{\text{BRS}} A^0(y)) | 0 \rangle = 0, \quad (\text{A.1})$$

with

$$\delta_{\text{BRS}} A^0 = -\frac{g}{2}(c^+ H^- + c^- H^+) + \frac{e}{s_{2W}} c^Z (c_{\alpha-\beta} h^0 + s_{\alpha-\beta} H^0), \quad (\text{A.2})$$

$$\text{and } \delta_{\text{BRS}} \bar{c}^Z = B^Z. \quad (\text{A.3})$$

Therefore,

$$\begin{aligned} & \langle 0 | B^Z(x) A^0(y) | 0 \rangle + \frac{g}{2} \left( \langle 0 | \bar{c}^Z(x) c^+(y) H^-(y) | 0 \rangle + \langle 0 | \bar{c}^Z(x) c^-(y) H^+(y) | 0 \rangle \right) \\ & - \frac{e}{s_{2W}} \left( c_{\alpha-\beta} \langle 0 | \bar{c}^Z(x) c^Z(y) h^0(y) | 0 \rangle + s_{\alpha-\beta} \langle 0 | \bar{c}^Z(x) c^Z(y) H^0(y) | 0 \rangle \right) = 0. \end{aligned} \quad (\text{A.4})$$

At tree-level, there is no vertex involving  $\bar{c}^Z c^\pm H^\pm$ . Using the equation of motion of the B field, we obtain a relation for the following Green’s functions (external legs are not amputated):

$$\begin{aligned} & \partial_x \langle 0 | Z^0(x) A^0(y) | 0 \rangle + M_{Z^0} \langle 0 | G^0(x) A^0(y) | 0 \rangle \\ & + \frac{e}{s_{2W}} \left( \tilde{\epsilon} \langle 0 | h^0(x) G^0(x) A^0(y) | 0 \rangle + \tilde{\gamma} \langle 0 | H^0(x) G^0(x) A^0(y) | 0 \rangle \right) \\ & + \frac{e}{s_{2W}} \left( c_{\alpha-\beta} \langle 0 | \bar{c}^Z(x) c^Z(y) h^0(y) | 0 \rangle + s_{\alpha-\beta} \langle 0 | \bar{c}^Z(x) c^Z(y) H^0(y) | 0 \rangle \right) = 0. \end{aligned} \quad (\text{A.5})$$

In a diagrammatic form, we have

$$\begin{aligned} & \frac{1}{q^2 - M_{Z^0}^2} \frac{1}{q^2 - M_{A^0}^2} \left( i q_\mu \times Z^\mu \dashrightarrow \odot \dashrightarrow A^0 + M_{Z^0} \times G^0 \dashrightarrow \odot \dashrightarrow A^0 \right) \\ & = -\frac{i}{q^2 - M_{A^0}^2} \frac{e}{s_{2W}} \left( \tilde{\epsilon} \times \odot_{h^0}^{G^0} \dashrightarrow A^0 + \tilde{\gamma} \times \odot_{H^0}^{G^0} \dashrightarrow A^0 \right) \\ & + \frac{i}{q^2 - M_{Z^0}^2} \frac{e}{s_{2W}} \left( c_{\alpha-\beta} \times \bar{c}^Z \dashrightarrow \odot_{h^0}^{c^Z} + s_{\alpha-\beta} \times \bar{c}^Z \dashrightarrow \odot_{H^0}^{c^Z} \right), \end{aligned} \quad (\text{A.6})$$

and obtain the relation

$$\begin{aligned} & q^2 \Sigma_{A^0 Z^0}(q^2) + M_{Z^0} \Sigma_{A^0 G^0}(q^2) = -(q^2 - M_{Z^0}^2) \frac{i e}{s_{2W}} \left( \tilde{\epsilon} \times \odot_{h^0}^{G^0} \dashrightarrow A^0 + \tilde{\gamma} \times \odot_{H^0}^{G^0} \dashrightarrow A^0 \right) \\ & + (q^2 - M_{A^0}^2) \frac{i e}{s_{2W}} \left( c_{\alpha-\beta} \times \bar{c}^Z \dashrightarrow \odot_{h^0}^{c^Z} + s_{\alpha-\beta} \times \bar{c}^Z \dashrightarrow \odot_{H^0}^{c^Z} \right). \end{aligned} \quad (\text{A.7})$$

With the following vertices

$$\mathcal{L} \supset -\frac{e M_{Z^0}}{s_{2W}} s_{2\beta} \left( s_{\alpha+\beta} h^0 - c_{\alpha+\beta} H^0 \right) A^0 G^0, \quad (\text{A.8})$$

$$\mathcal{L}^{Gh} \supset \frac{e M_{Z^0}}{s_{2W}} \left( (s_{\alpha-\beta} - \tilde{\epsilon}) h^0 - (c_{\alpha-\beta} + \tilde{\gamma}) H^0 \right) \bar{c}^Z c^Z, \quad (\text{A.9})$$

we calculate all the “lollipops”

$$\odot_{h^0}^{G^0} \dashrightarrow A^0 = -i \frac{e M_{Z^0}}{s_{2W}} s_{2\beta} s_{\alpha+\beta} B_0(q^2, M_{h^0}^2, M_{Z^0}^2), \quad (\text{A.10})$$

$$\odot_{H^0}^{G^0} \dashrightarrow A^0 = i \frac{e M_{Z^0}}{s_{2W}} s_{2\beta} c_{\alpha+\beta} B_0(q^2, M_{H^0}^2, M_{Z^0}^2), \quad (\text{A.11})$$

$$\bar{c}^Z \dashrightarrow \odot_{h^0}^{c^Z} = i \frac{e M_{Z^0}}{s_{2W}} (s_{\alpha-\beta} - \tilde{\epsilon}) B_0(q^2, M_{h^0}^2, M_{Z^0}^2), \quad (\text{A.12})$$

$$\bar{c}^Z \dashrightarrow \odot_{H^0}^{c^Z} = -i \frac{e M_{Z^0}}{s_{2W}} (c_{\alpha-\beta} + \tilde{\gamma}) B_0(q^2, M_{H^0}^2, M_{Z^0}^2), \quad (\text{A.13})$$

with

$$B_0(q^2, M_1^2, M_2^2) = C_{UV} - \int_0^1 dx \ln(\Delta(q^2, M_1^2, M_2^2)), \quad (\text{A.14})$$

$$\Delta(q^2, M_1^2, M_2^2) = q^2 x^2 - (q^2 + M_2^2 - M_1^2)x + M_2^2. \quad (\text{A.15})$$

We finally obtain the identity

$$q^2 \Sigma_{A^0 Z^0}(q^2) + M_{Z^0} \Sigma_{A^0 G^0}(q^2) = \frac{1}{(4\pi)^2} \frac{e^2 M_{Z^0}}{s_{2W}^2} \left( (q^2 - M_{Z^0}^2) s_{2\beta} \mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) + (q^2 - M_{A^0}^2) \mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) \right),$$

with  $\mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) = \tilde{\gamma} c_{\alpha+\beta} B_0(q^2, M_{H^0}^2, M_{Z^0}^2) - \tilde{\epsilon} s_{\alpha+\beta} B_0(q^2, M_{h^0}^2, M_{Z^0}^2),$   
 $\mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) = \tilde{\epsilon} c_{\alpha-\beta} B_0(q^2, M_{h^0}^2, M_{Z^0}^2) + \tilde{\gamma} s_{\alpha-\beta} B_0(q^2, M_{H^0}^2, M_{Z^0}^2)$   
 $+ \frac{1}{2} s_{2(\alpha-\beta)} (B_0(q^2, M_{H^0}^2, M_{Z^0}^2) - B_0(q^2, M_{h^0}^2, M_{Z^0}^2)).$  (A.16)

To implement this formula into **SloopS** and check it numerically, we need to introduce the tadpole part in **FormCalc** and we define  $\Sigma^{\text{tad}}$  the self-energy without tadpole:

$$q^2 \Sigma_{A^0 Z^0}^{\text{tad}}(q^2) + M_{Z^0} \Sigma_{A^0 G^0}^{\text{tad}}(q^2) + M_{Z^0} \delta T = \frac{1}{(4\pi)^2} \frac{e^2 M_{Z^0}}{s_{2W}^2} ((q^2 - M_{Z^0}^2) s_{2\beta} \mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}} + (q^2 - M_{A^0}^2) \mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}),$$

where  $\delta T = \frac{e}{s_{2W} M_{Z^0}} (s_{\alpha-\beta} \delta T_{H^0} + c_{\alpha-\beta} \delta T_{h^0}).$  (A.17)

We remark on some simplifications in the functions  $\mathcal{F}$  for specific choices of the non linear gauge parameters

$$\mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(\tilde{\epsilon} = 0, \tilde{\gamma} = 0) = 0, \quad (\text{A.18})$$

$$\mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(\tilde{\epsilon} = c_{\alpha+\beta}, \tilde{\gamma} = s_{\alpha+\beta}) = \frac{1}{2} s_{2(\alpha+\beta)} \int_0^1 dx \ln \left( \frac{\Delta(q^2, M_{h^0}^2, M_{Z^0}^2)}{\Delta(q^2, M_{H^0}^2, M_{Z^0}^2)} \right), \quad (\text{A.19})$$

$$\mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}(\tilde{\epsilon} = s_{\alpha-\beta}, \tilde{\gamma} = -c_{\alpha-\beta}) = 0, \quad (\text{A.20})$$

$$\mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}(\tilde{\epsilon} = 0, \tilde{\gamma} = 0) = \frac{1}{2} s_{2(\alpha-\beta)} \int_0^1 dx \ln \left( \frac{\Delta(q^2, M_{h^0}^2, M_{Z^0}^2)}{\Delta(q^2, M_{H^0}^2, M_{Z^0}^2)} \right). \quad (\text{A.21})$$

In terms of renormalised self energies,

$$\hat{\Sigma}_{A^0 Z^0}(q^2) = \Sigma_{A^0 Z^0}^{\text{tad}}(q^2) + \frac{M_{Z^0}}{2} (\delta Z_{G^0 A^0} + s_{2\beta} \frac{\delta t_\beta}{t_\beta}), \quad (\text{A.22})$$

$$\hat{\Sigma}_{A^0 G^0}(q^2) = \Sigma_{A^0 G^0}^{\text{tad}}(q^2) + \delta M_{A^0 G^0}^2 - \frac{1}{2} q^2 \delta Z_{G^0 A^0} - \frac{1}{2} (q^2 - M_{A^0}^2) \delta Z_{A^0 G^0}, \quad (\text{A.23})$$

with (Eq. 4.18)

$$\delta M_{A^0 G^0}^2 = \delta T - \frac{1}{2} s_{2\beta} M_{A^0}^2 \frac{\delta t_\beta}{t_\beta}, \quad (\text{A.24})$$

we obtain the following constraint on the *renormalised* two-point functions

$$\begin{aligned} q^2 \hat{\Sigma}_{A^0 Z^0}(q^2) + M_{Z^0} \hat{\Sigma}_{A^0 G^0}(q^2) &= (q^2 - M_{Z^0}^2) \frac{1}{(4\pi)^2} \frac{e^2 M_{Z^0}}{s_{2W}^2} s_{2\beta} \mathcal{F}_{GA}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) \\ &+ \frac{M_{Z^0}}{2} (q^2 - M_{A^0}^2) \left( \frac{1}{(4\pi)^2} \frac{2e^2}{s_{2W}^2} \mathcal{F}_{cc}^{\tilde{\epsilon}, \tilde{\gamma}}(q^2) + s_{2\beta} \frac{\delta t_\beta}{t_\beta} - \delta Z_{AG} \right). \end{aligned} \quad (\text{A.25})$$

Note that in this identity  $\delta T$  and more importantly  $\delta Z_{G^0 A^0}$  drop out.

The derivation of the identity for the charged Higgses follows along the same steps. We only quote the result

$$\begin{aligned} q^2 \hat{\Sigma}_{H^+ W^+}(q^2) + M_{W^\pm} \hat{\Sigma}_{H^+ W^+}(q^2) &= (q^2 - M_{W^\pm}^2) \frac{1}{(4\pi)^2} \frac{e^2 M_{W^\pm}}{s_{2W}^2} \mathcal{G}_{HW}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2) \\ &+ \frac{M_{W^\pm}}{2} (q^2 - M_{H^\pm}^2) \left( \frac{1}{(4\pi)^2} \frac{2e^2}{s_{2W}^2} \mathcal{G}_{cc}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2) + s_{2\beta} \frac{\delta t_\beta}{t_\beta} - \delta Z_{H^\pm G^\pm} \right). \end{aligned}$$

with the functions  $\mathcal{G}_{HW}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2)$  and  $\mathcal{G}_{cc}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2)$  defined as

$$\begin{aligned} \mathcal{G}_{HW}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2) &= \tilde{\delta} (s_{2\beta} s_{\alpha+\beta} - c_W^2 c_{\alpha-\beta}) B_0(q^2, M_{W^\pm}^2, M_{h^0}^2) - \tilde{\omega} (c_{2\beta} s_{\alpha+\beta} - s_W^2 s_{\alpha-\beta}) B_0(q^2, M_{W^\pm}^2, M_{H^0}^2) \\ &+ \tilde{\rho} c_W^2 B_0(q^2, M_{W^\pm}^2, M_{A^0}^2), \\ \mathcal{G}_{cc}^{\tilde{\rho}, \tilde{\omega}, \tilde{\delta}}(q^2) &= c_{\alpha-\beta} (s_{\alpha-\beta} - \tilde{\delta}) c_W^2 B_0(q^2, M_{W^\pm}^2, M_{h^0}^2) - s_{\alpha-\beta} (c_{\alpha-\beta} + \tilde{\omega}) c_W^2 B_0(q^2, M_{W^\pm}^2, M_{H^0}^2) \\ &- \tilde{\rho} c_W^2 B_0(q^2, M_{W^\pm}^2, M_{A^0}^2). \end{aligned} \quad (\text{A.26})$$

## B Wave function renormalisation constants before rotation

In our approach field renormalisation was performed on the physical fields, or better said, after rotation to the  $h^0, H^0, A^0, G^0, H^\pm, G^\pm$  basis. We could have applied field renormalisation on the components of the doublets  $H_1, H_2$ , Eq. (2.3). To make contact with some of the early papers [14, 15, 19] on the renormalisation of the Higgs sector we therefore introduce the most general field renormalisation on the components of  $H_1, H_2$ . We define

$$\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}_0 = \begin{pmatrix} Z_{\varphi_1^0}^{1/2} & Z_{\varphi_1^0 \varphi_2^0}^{1/2} \\ Z_{\varphi_2^0 \varphi_1^0}^{1/2} & Z_{\varphi_2^0}^{1/2} \end{pmatrix} \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}, \quad (\text{B.1})$$

$$\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}_0 = \begin{pmatrix} Z_{\phi_1^\pm}^{1/2} & Z_{\phi_1^\pm \phi_2^\pm}^{1/2} \\ Z_{\phi_2^\pm \phi_1^\pm}^{1/2} & Z_{\phi_2^\pm}^{1/2} \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \quad (\text{B.2})$$

$$\begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}_0 = \begin{pmatrix} Z_{\phi_1^0}^{1/2} & Z_{\phi_1^0 \phi_2^0}^{1/2} \\ Z_{\phi_2^0 \phi_1^0}^{1/2} & Z_{\phi_2^0}^{1/2} \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}. \quad (\text{B.3})$$

As explained in the text, these constants are immediately transformed into the set of matrices  $Z_P, Z_C, Z_S$ . Or we can go from the set  $Z_P, Z_C, Z_S$  to the set defined by Eqs. (B.1-B.3). For

example,

$$\begin{cases} \delta Z_{G^0} = c_\beta^2 \delta Z_{\varphi_1^0} + s_\beta^2 \delta Z_{\varphi_2^0} + c_\beta s_\beta (\delta Z_{\varphi_1^0 \varphi_2^0} + \delta Z_{\varphi_2^0 \varphi_1^0}) \\ \delta Z_{G^0 A^0} = c_\beta s_\beta (\delta Z_{\varphi_2^0} - \delta Z_{\varphi_1^0}) + c_\beta^2 \delta Z_{\varphi_1^0 \varphi_2^0} - s_\beta^2 \delta Z_{\varphi_2^0 \varphi_1^0} \\ \delta Z_{A^0 G^0} = c_\beta s_\beta (\delta Z_{\varphi_2^0} - \delta Z_{\varphi_1^0}) + c_\beta^2 \delta Z_{\varphi_2^0 \varphi_1^0} - s_\beta^2 \delta Z_{\varphi_1^0 \varphi_2^0} \\ \delta Z_{A^0} = s_\beta^2 \delta Z_{\varphi_1^0} + c_\beta^2 \delta Z_{\varphi_2^0} - c_\beta s_\beta (\delta Z_{\varphi_1^0 \varphi_2^0} + \delta Z_{\varphi_2^0 \varphi_1^0}) \end{cases} \quad (\text{B.4})$$

$$\begin{cases} \delta Z_{G^\pm} = c_\beta^2 \delta Z_{\phi_1^\pm} + s_\beta^2 \delta Z_{\phi_2^\pm} + c_\beta s_\beta (\delta Z_{\phi_1^\pm \phi_2^\pm} + \delta Z_{\phi_2^\pm \phi_1^\pm}) \\ \delta Z_{G^\pm H^\pm} = c_\beta s_\beta (\delta Z_{\phi_2^\pm} - \delta Z_{\phi_1^\pm}) + c_\beta^2 \delta Z_{\phi_1^\pm \phi_2^\pm} - s_\beta^2 \delta Z_{\phi_2^\pm \phi_1^\pm} \\ \delta Z_{H^\pm G^\pm} = c_\beta s_\beta (\delta Z_{\phi_2^\pm} - \delta Z_{\phi_1^\pm}) + c_\beta^2 \delta Z_{\phi_2^\pm \phi_1^\pm} - s_\beta^2 \delta Z_{\phi_1^\pm \phi_2^\pm} \\ \delta Z_{H^\pm} = s_\beta^2 \delta Z_{\phi_1^\pm} + c_\beta^2 \delta Z_{\phi_2^\pm} - c_\beta s_\beta (\delta Z_{\phi_1^\pm \phi_2^\pm} + \delta Z_{\phi_2^\pm \phi_1^\pm}) \end{cases} \quad (\text{B.5})$$

$$\begin{cases} \delta Z_{H^0} = c_\alpha^2 \delta Z_{\phi_1^0} + s_\alpha^2 \delta Z_{\phi_2^0} + c_\alpha s_\alpha (\delta Z_{\phi_1^0 \phi_2^0} + \delta Z_{\phi_2^0 \phi_1^0}) \\ \delta Z_{H^0 h^0} = c_\alpha s_\alpha (\delta Z_{\phi_2^0} - \delta Z_{\phi_1^0}) + c_\alpha^2 \delta Z_{\phi_1^0 \phi_2^0} - s_\alpha^2 \delta Z_{\phi_2^0 \phi_1^0} \\ \delta Z_{h^0 H^0} = c_\alpha s_\alpha (\delta Z_{\phi_2^0} - \delta Z_{\phi_1^0}) + c_\alpha^2 \delta Z_{\phi_2^0 \phi_1^0} - s_\alpha^2 \delta Z_{\phi_1^0 \phi_2^0} \\ \delta Z_{h^0} = s_\alpha^2 \delta Z_{\phi_1^0} + c_\alpha^2 \delta Z_{\phi_2^0} - c_\alpha s_\alpha (\delta Z_{\phi_1^0 \phi_2^0} + \delta Z_{\phi_2^0 \phi_1^0}) \end{cases} \quad (\text{B.6})$$

$$\begin{cases} \delta Z_{H^0 h^0} + \delta Z_{h^0 H^0} = (c_\alpha^2 - s_\alpha^2) (\delta Z_{\phi_1^0 \phi_2^0} + \delta Z_{\phi_2^0 \phi_1^0}) + 2c_\alpha s_\alpha (\delta Z_{\phi_2^0} - \delta Z_{\phi_1^0}) \\ \delta Z_{H^0 h^0} - \delta Z_{h^0 H^0} = \delta Z_{\phi_1^0 \phi_2^0} - \delta Z_{\phi_2^0 \phi_1^0} \end{cases} \quad (\text{B.7})$$

Our renormalisation conditions in Eq. (4.36) on  $\hat{\Sigma}_{ii}$  will turn into

$$Re\Sigma'_{A^0 A^0}(M_{A^0}^2) = s_\beta^2 \delta Z_{\varphi_1^0} + c_\beta^2 \delta Z_{\varphi_2^0} - c_\beta s_\beta (\delta Z_{\varphi_1^0 \varphi_2^0} + \delta Z_{\varphi_2^0 \varphi_1^0}), \quad (\text{B.8})$$

$$Re\Sigma'_{H^\pm H^\pm}(M_{H^\pm}^2) = s_\beta^2 \delta Z_{\phi_1^\pm} + c_\beta^2 \delta Z_{\phi_2^\pm} - c_\beta s_\beta (\delta Z_{\phi_1^\pm \phi_2^\pm} + \delta Z_{\phi_2^\pm \phi_1^\pm}), \quad (\text{B.9})$$

$$Re\Sigma'_{H^0 H^0}(M_{H^0}^2) = c_\alpha^2 \delta Z_{\phi_1^0} + s_\alpha^2 \delta Z_{\phi_2^0} + c_\alpha s_\alpha (\delta Z_{\phi_1^0 \phi_2^0} + \delta Z_{\phi_2^0 \phi_1^0}), \quad (\text{B.10})$$

$$Re\Sigma'_{h^0 h^0}(M_{h^0}^2) = s_\alpha^2 \delta Z_{\phi_1^0} + c_\alpha^2 \delta Z_{\phi_2^0} - c_\alpha s_\alpha (\delta Z_{\phi_1^0 \phi_2^0} + \delta Z_{\phi_2^0 \phi_1^0}). \quad (\text{B.11})$$

In fact in [14, 15, 19] only two renormalisation constants are introduced, one for each doublet through

$$H_i \rightarrow (1 + \frac{1}{2} \delta Z_{H_i}) H_i \quad i = 1, 2. \quad (\text{B.12})$$

This means that

$$\begin{aligned} \delta Z_{\phi_i^0} &= \delta Z_{\varphi_i^0} = \delta Z_{\phi_i^\pm} = \delta Z_{H_i}, \\ \delta Z_{\phi_i^0 \phi_j^0} &= \delta Z_{\varphi_i^0 \varphi_j^0} = \delta Z_{\phi_i^\pm \phi_j^\pm} = 0 \quad i \neq j. \end{aligned} \quad (\text{B.13})$$

Since wave function renormalisation is applied on the doublets it also contributes a shift to  $v_i$ . Another shift on this parameter is also applied,  $v_i \rightarrow v_i - \tilde{\delta} v_i$  as to all other Lagrangian parameters. Compared to our shift  $\delta v_i$ , we have

$$\delta v_i = \tilde{\delta} v_i - \frac{1}{2} \delta Z_{H_i} v_i. \quad (\text{B.14})$$

Note that with only  $\delta Z_{H_1}$  and  $\delta Z_{H_2}$ , in view of Eqs. (B.10)-(B.11) and Eq. (B.13) we have

$$\delta Z_{H_1} - \delta Z_{H_2} = -\frac{1}{2c_{2\alpha}} \left( Re\Sigma'_{h^0 h^0}(M_{h^0}^2) - Re\Sigma'_{H^0 H^0}(M_{H^0}^2) \right). \quad (\text{B.15})$$



## References

- [1] Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1;  
*ibid. Phys. Lett.* **B262** (1991) 54;  
 J.R. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B25** (1991) 83;  
*ibid. Phys. Lett.* **B262** (1991) 477;  
 H.E. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815;  
 A. Yamada, *Phys. Lett.* **B263** (1991) 233.  
 There is a long list of papers on the one-loop corrections to the lightest Higgs mass. It is fair to add to the list the seminal paper, written with the assumption that the top is not heavier than 60 GeV:  
 S.P. Li and M. Sher, *Phys. Lett.* **B140** (1984) 339.
- [2] A. Djouadi, *Phys. Rept.* **459** (2008) 1, hep-ph/0503173.
- [3] S.P. Martin, hep-ph/9709356.
- [4] For a review on the corrections at the two-loop order, see for example  
 S.P. Martin, *Phys. Rev.* **D71** (2005) 016012, hep-ph/0405022.
- [5] S.P. Martin, *Phys. Rev.* **D75** (2007) 055005, hep-ph/0701051;  
 R.V. Harlander, P. Kant, L. Mihaila and M. Steinhauser, *Phys. Rev. Lett.* **100** (2008) 191602, arXiv:0803.0672 [hep-ph].
- [6] A. Freitas and D. Stöckinger, *Phys. Rev.* **D66** (2002) 095014, hep-ph/0205281.
- [7] Y. Yamada, *Phys. Lett.* **B530** (2002) 174, hep-ph/0112251.
- [8] S. Davidson and H.E. Haber, *Phys. Rev.* **D72** (2005) 035004, [Erratum-*ibid.* **D72** (2005) 099902], hep-ph/0504050.
- [9] G. Bélanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato and Y. Shimizu, *Phys. Rep.* **430** (2006) 117, hep-ph/0308080.
- [10] F. Boudjema and E. Chopin, *Z. Phys.* **C73** (1996) 85, hep-ph/9507396.
- [11] F. Boudjema, A. Semenov and D. Temes, *Phys. Rev.* **D72** (2005) 055024, hep-ph/0507127.
- [12] N. Baro, F. Boudjema and A. Semenov, *Phys. Lett.* **B660** (2008) 550, arXiv:0710.1821 [hep-ph].
- [13] J. Fujimoto, T. Ishikawa, Y. Kurihara, M. Jimbo, T. Kon and M. Kuroda, *Phys. Rev.* **D75** (2007) 113002.
- [14] A. Dabelstein, *Z. Phys.* **C67** (1995) 495, hep-ph/9409375.
- [15] P.H. Chankowski, S. Pokorski and J. Rosiek, *Nucl. Phys.* **B423** (1994) 437, hep-ph/9303309.
- [16] A. Dabelstein, *Nucl. Phys.* **B456** (1995) 25, hep-ph/9503443.
- [17] L.Y. Shan, *Eur. Phys. J.* **C12** (2000) 113, hep-ph/9807456.
- [18] H.E. Logan and S.F. Su, *Phys. Rev.* **D66** (2002) 035001, hep-ph/0203270.
- [19] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rept.* **425** (2006) 265, hep-ph/0412214.
- [20] D. Pierce and A. Papadopoulos, *Phys. Rev.* **D47** (1993) 222, hep-ph/9206257.
- [21] F. Boudjema and A. Semenov, *Phys. Rev.* **D66** (2002) 095007, hep-ph/0201219.
- [22] M. Drees and K.I. Hikasa, *Phys. Lett.* **B240** (1990) 455, [Erratum-*ibid.* **B262** (1991) 497].
- [23] N. Baro and F. Boudjema, paper in draft form, to appear soon.
- [24] G. Bélanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato and Y. Shimizu, *Phys. Lett.* **B559** (2003) 252, hep-ph/0212261;  
 G. Bélanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato and Y. Shimizu, *Phys. Lett.* **B559** (2003) 252, hep-ph/0212261;  
 G. Bélanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Shimizu and Y. Yasui, *Phys. Lett.* **B571** (2003) 163, hep-ph/0307029;

- G. Bélanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, Y. Kurihara, K. Kato and Y. Shimizu, *Phys. Lett.* **B576** (2003) 152, hep-ph/0309010;  
 F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Kurihara, Y. Shimizu and Y. Yasui, *Phys. Lett.* **B600** (2004) 65, hep-ph/0407065;  
 F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Kurihara, Y. Shimizu, S. Yamashita and Y. Yasui, *Nucl. Instrum and Meth.* **A534** (2004) 334, hep-ph/0404098.
- [25] A. Denner, S. Dittmaier, M. Roth and M.M. Weber, *Phys. Lett.* **B575** (2003) 290, hep-ph/0307193;  
 A. Denner, S. Dittmaier, M. Roth and M.M. Weber, *Phys. Lett.* **B560** (2003) 196, hep-ph/0301189;  
 You Yu, Ma Wen-Gan, Chen Hui, Zhang Ren-You, Sun Yan-Bin and Hou Hong-Sheng, *Phys. Lett.* **B571** (2003) 85, hep-ph/0306036;  
 Zhang Ren-You, Ma Wen-Gan, Chen Hui, Sun Yan-Bin, Hou Hong-Sheng, *Phys. Lett.* **B578** (2004) 349, hep-ph/0308203.
- [26] F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Kurihara and Y. Shimizu, *Nucl. Phys. Proc. Suppl.* **135** (2004) 323, hep-ph/0407079;  
 F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Kurihara, Y. Shimizu and Y. Yasui, hep-ph/0510184.
- [27] A. Denner, S. Dittmaier, M. Roth and L.H. Wieders, *Phys. Lett.* **B612** (2005) 223, hep-ph/0502063;  
 A. Denner, S. Dittmaier, M. Roth and L.H. Wieders, hep-ph/0505042.
- [28] J. Küblbeck, M. Böhm, and A. Denner, *Comp. Phys. Commun.* **60** (1990) 165;  
 H. Eck and J. Küblbeck, *Guide to FeynArts 1.0*, Würzburg, 1991;  
 H. Eck, *Guide to FeynArts 2.0*, Würzburg, 1995;  
 T. Hahn, *Comp. Phys. Commun.* **140** (2001) 418, hep-ph/0012260.
- [29] T. Hahn and M. Perez-Victoria, *Comp. Phys. Commun.* **118** (1999) 153, hep-ph/9807565;  
 T. Hahn, hep-ph/0406288; hep-ph/0506201.
- [30] T. Hahn, LoopTools, <http://www.feynarts.de/looptools/>.
- [31] T. Hahn and C. Schappacher, *Comp. Phys. Commun.* **143** (2002) 54, hep-ph/0105349.
- [32] J. Fujimoto *et al.*, *Comput. Phys. Commun.* **153** (2003) 106, hep-ph/0208036.
- [33] A. Semenov, hep-ph/9608488;  
 A. Semenov, *Nucl. Inst. Meth. and Inst.* **A393** (1997) 293;  
 A. Semenov, *Comp. Phys. Commun.* **115** (1998) 124;  
 A. Semenov, hep-ph/0208011;  
 A. Semenov, arXiv:0805.0555 [hep-ph].
- [34] [CompHEP Collaboration], E. Boos *et al.*, *Nucl. Instrum. Meth.* **A534** (2004) 250, hep-ph/0403113;  
 A. Pukhov *et al.*, "CompHEP user's manual, v3.3", Preprint INP MSU 98-41/542 (1998) hep-ph/9908288;  
<http://theory.sinp.msu.ru/comphep/>.
- [35] M.S. Carena, S. Heinemeyer, C.E.M. Wagner and G. Weiglein, hep-ph/9912223.
- [36] [ATLAS/CMS Collaboration], Martin Flechl *et al.*, ATL-PHYS-CONF-2007-018, arXiv:0710.1761v4 [hep-ph].